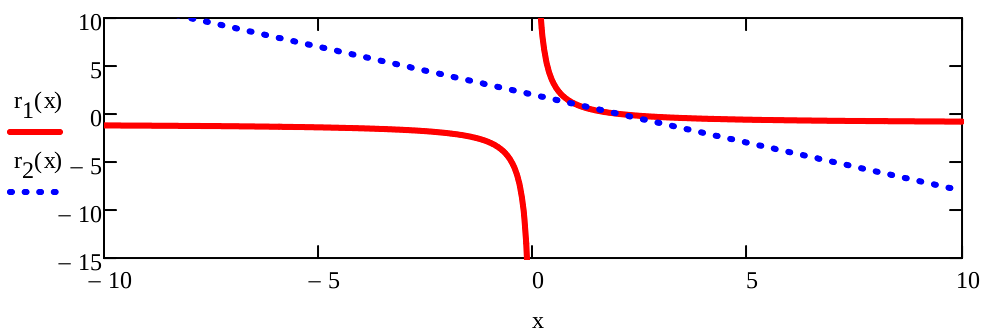
Worksheet on Mathematical Modeling in Open-Equation Format

1. Rearrange to equation in residual form to:
    - a. Avoid divide by zero
    - b. Minimize use of functions like sqrt, log, exp, etc.
    - b. Have continuous first and second derivatives
    - c. Fit the equation into a linear or quadratic form
2. Bounds
    - a. Include variable bounds to exclude infeasible solutions
    - b. Variable bounds to avoid regions of strong nonlinearity
    - c. Caution: watch for infeasible solutions
2. Scaling:
    - a. Scale absolute value of variables to 1e-3 to 1e3
    - b. Scale absolute value of equation residuals to 1e-3 to 1e3
    - c. Better that 1st derivative values are closer to 1.0
3. Good initial conditions:
    - a. Starting near a solution can improve convergence
    - b. Try multiple initial conditions to verify global solution (non-convex problems)
    - c. Explicitly calculate intermediate values
4. Check iteration summary for improved convergence

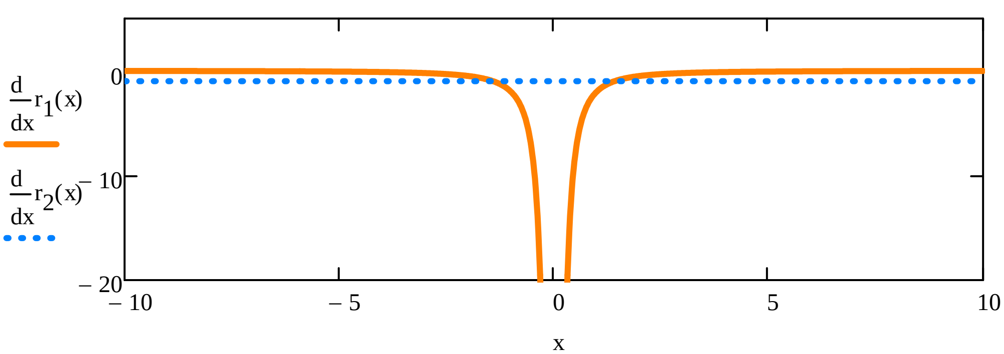
Exercise 1: Bad Good

$x = 2$ $\qquad r_1(x) := \frac{2}{x} - 1$ $\qquad r_2(x) := 2 - x$

Residuals



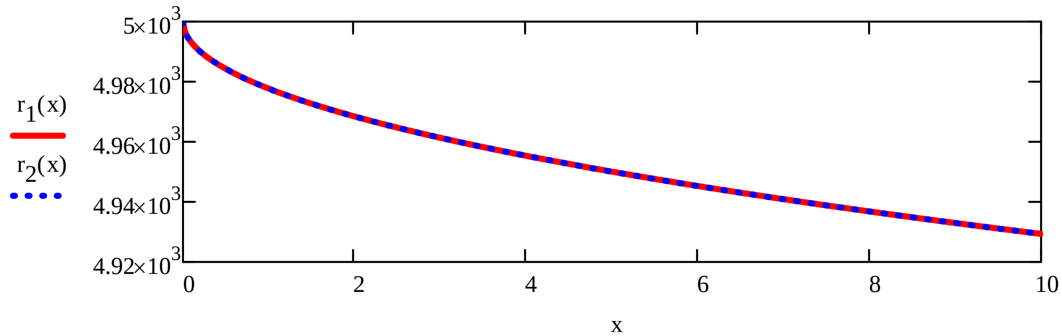

1st Derivative



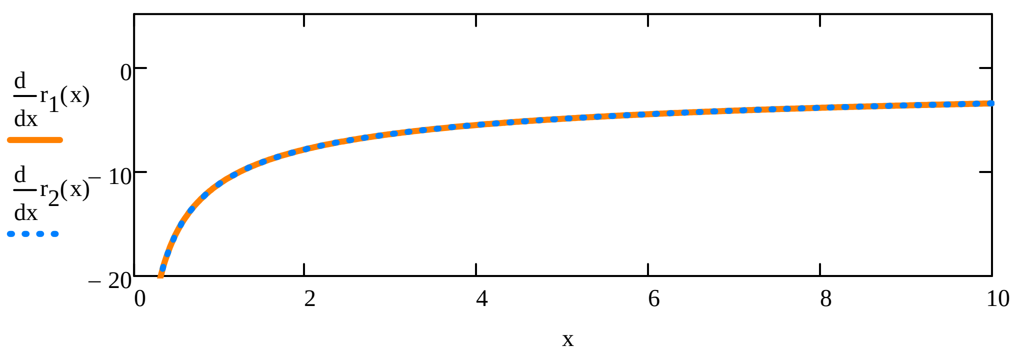
Exercise 2: $5000 = \sqrt{500 \cdot x}$ Rearrange / scale equation for improved performance

$r_1(x) := 5000 - \sqrt{500 \cdot x}$ $r_2(x) := r_1(x)$ $r_2(x) =$

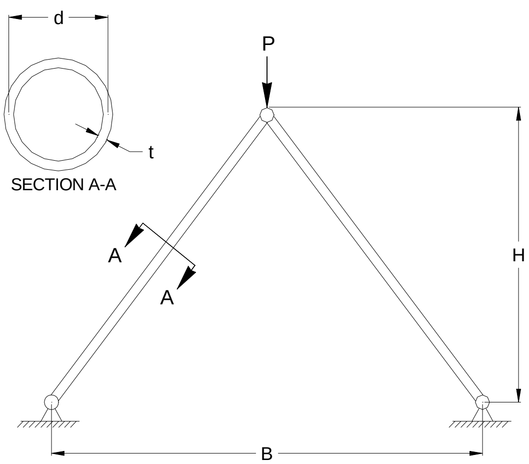
Residuals




1st Derivative



Exercise 3:




$Height := 30 \cdot in$

$Diameter := 3.0 \cdot in$

$Thickness := 0.15 \cdot in$

$Separation := 60 \cdot in$

$Modulus := 3 \cdot 10^{7} \cdot \frac{lbf}{in^{2}}$

$Density := 0.3 \cdot \frac{lbm}{in^{3}}$

$Load := 66 \cdot 10^{3} \cdot lbf$

$\rho := Density$ $\quad d := Diameter$ $\quad t := Thickness$ $\quad B := Separation$ $\quad H := Height$

$P := Load$ $\quad E := Modulus$

$$Weight := \rho \cdot 2 \cdot \pi \cdot d \cdot t \cdot \sqrt{\left(\frac{B}{2}\right)^{2} + H^{2}} \qquad Weight = 35.987 \cdot lb$$

$$Stress := \frac{P \cdot \sqrt{\left(\frac{B}{2}\right)^{2} + H^{2}}}{2 \cdot t \cdot \pi \cdot d \cdot H} \qquad Stress = 33.012 \cdot ksi$$

$$Buckling\_Stress := \frac{\pi^{2} \cdot E \cdot \left(d^{2} + t^{2}\right)}{8 \cdot \left[\left(\frac{B}{2}\right)^{2} + H^{2}\right]} \qquad Buckling\_Stress = 185.518 \cdot ksi$$

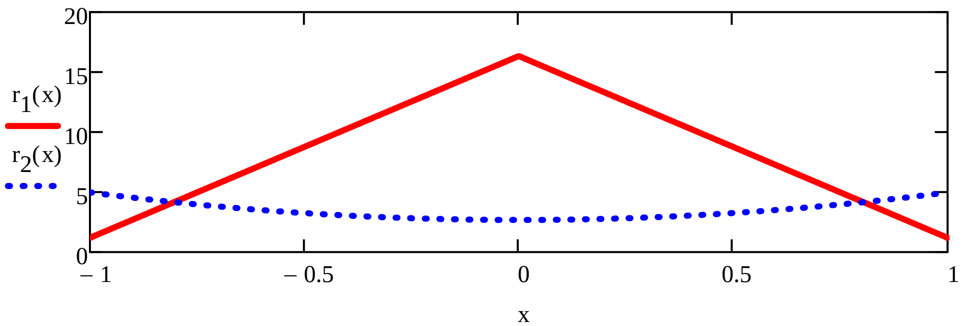
$$Deflection := \frac{P \cdot \left[\left(\frac{B}{2}\right)^{2} + H^{2}\right]^{\frac{3}{2}}}{2 \cdot t \cdot \pi \cdot d \cdot H^{2} \cdot E} \qquad Deflection = 0.066 \cdot in$$

$B := 0.1 \cdot in$

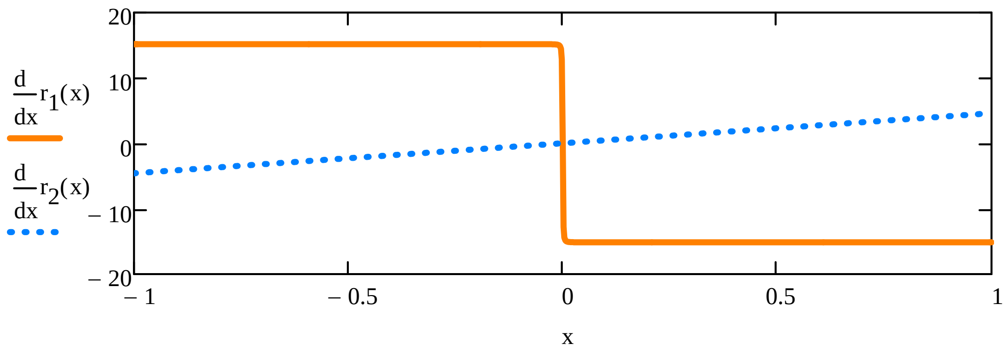
$r_1(x) := Weight - \rho \cdot 2 \cdot \pi \cdot d \cdot t \cdot \sqrt{\left(\frac{B}{2}\right)^2 + x^2}$

$r_2(x) := \frac{1}{100} \cdot \left[ Weight^2 + (\rho \cdot 2 \cdot \pi \cdot d \cdot t)^2 \cdot \left[ \left(\frac{B}{2}\right)^2 + x^2 \right] \right]$

Residuals




1st Derivative




Use of Constraints?

Weight > 0

Separation > 0

Etc