Unconstrained Optimization Worksheet

0. Background work: Practice and learn, on your own, the topics of Chapter3, Sections 1-3 in the Notes, specifically: does a search direction go uphill or downhill, positive and negative definiteness, necessary and sufficient conditions, representations of quadratic functions, and properties of quadratic functions.

1. For the function $f = x_1^2 - 2x_1x_2 + 4x_2^2$,

   a. Starting from the point x^T= [4,–1], execute <u>one</u> iteration only, by hand, of the method of steepest descent, using a line search with parabolic fit. Normalize the search direction. Start with an initial α=0.2.

   In a table list the trial values of **x**, α, f(**x**) for each step of the line search, as illustrated in the handout on steepest descent, as well as for the final values (**x***, $\alpha*$, f(**x***)) which result from the curve fit.

   b. Show search direction and the points of the line search on the accompanying contour plot.

   c. From the same starting point as in a) above, calculate $\alpha*$ using the formula, $\alpha^* = -\dfrac{\left(\nabla f^k\right)^T \mathbf{s}}{\mathbf{s}^T \mathbf{H}\mathbf{s}}$. Do the two alphas from a) and c) agree?

   d. From the same starting point as in a) above, execute one step of Newton's method.
   Show your calculations and show the resulting vector on the plot.
   Why did Newton's method drive to the optimum in one step?
   Using the criterion, $s^T \nabla f < 0$, show that Newton's method (in this instance) points downhill.

   e. Show that the necessary and sufficient conditions hold at the optimum to this function.

2. Solve the optimization problem with the following objective function with the associated MATLAB or Python scripts (see downloads):

   Minimize $x_1^4 - 2x_2x_1^2 + x_2^2 + x_1^2 - 2x_1 + 5$

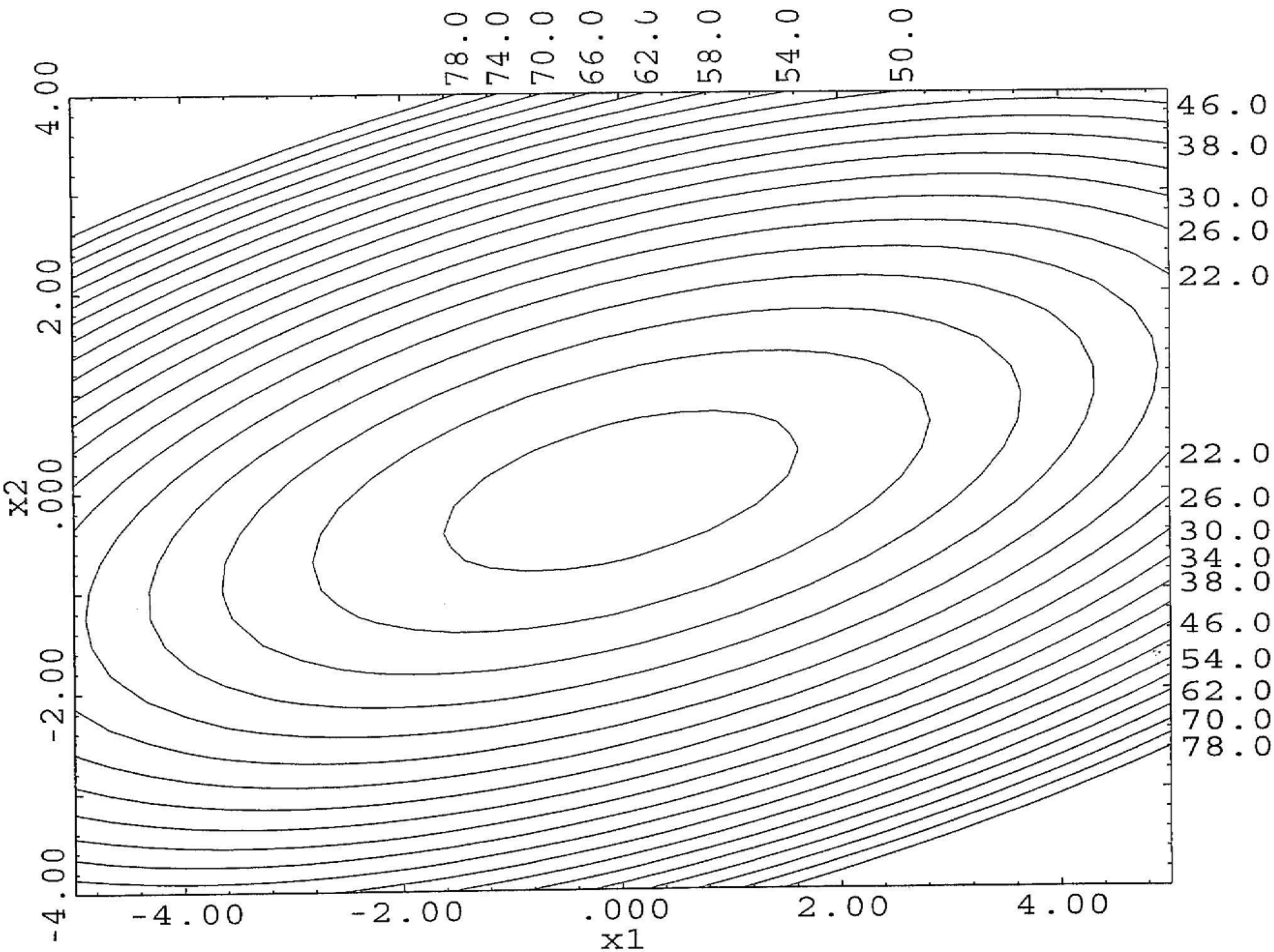
   Set the bounds for the variables to be –5 and +5. The starting point will be $\mathbf{x}^T$ = [1,4].

   a. Run 8 iterations of each algorithm (Newton's method, BFGS, Steepest Descent, Conjugate Gradient).
   b. For each iteration, record the values of x_1, x_2 and f and include in a table.
   c. Plot the values on a contour plot (either use the attached or create it with MATLAB or Python).
   d. Comment on and explain the progress of the algorithm.

3. For the function $f = x_1^2 - 2x_1x_2 + 4x_2^2$, (same as 1 above) and starting from the point x^T= [4,–1], compute the search direction for the <u>second</u> iteration of the quasi-Newton method using the BFGS update developed in class, giving the intermediate calculations. (Note, the first iteration will just be steepest descent, so take advantage of the work you did in the problem above.) Show the path of the search direction for the second iteration on the contour plot (start from the ending point of the first iteration from steepest descent).
   Does the search direction go thru the optimum? Why?

Note: Use MATLAB, Python, MathCAD or similar tool for computational purposes.

f = x1^2 -2x1x2 + 4x2^2



f =x1^4 - 2x2x1^2 + x2^2 + x1^2 - 2x1 + 5