Tsiolkovsky Rocket Optimization

Problem Statement

The Tsiolkovsky rocket [1] has a direct correlation between the change of velocity (Δv) of a rocket, wet mass (m_0), dry mass (m_f), and exhaust velocity (v_0) as shown in Equation 1.

$$\Delta v = v_0 \log \frac{m_0}{m_f} \tag{1}$$

This problem optimizes the design of a simple rocket for profit. Potential revenue increases with greater change in velocity, as greater velocities allow the payload to reach higher orbits that have less drag, allowing it to remain in orbit longer. Wet mass, dry mass, and exhaust velocity are design variables, where wet mass is the total initial mass of the rocket, including propellant, and dry mass is the mass of the rocket at full ascent.

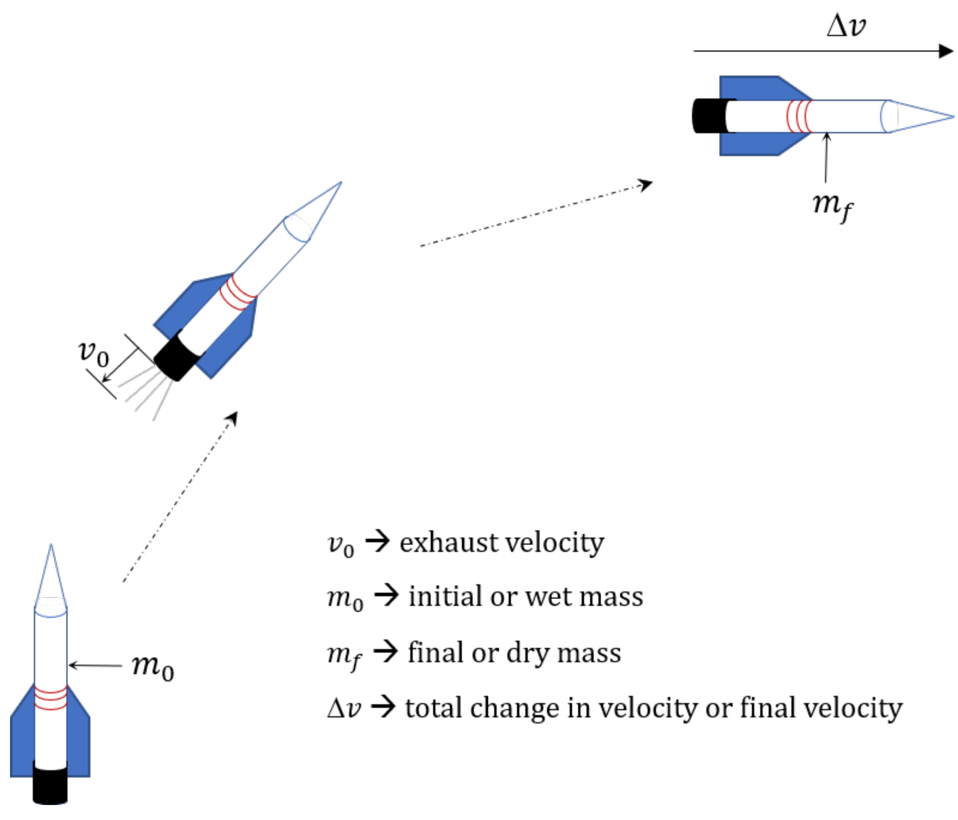

Figure 1: Rocket Launch

The rocket must have a dry mass of at least 20,000 kilograms and the change in velocity should be between 9400 meters per second and 20,200 meters per second. Varying designs allow for exhaust velocities ranging from 2,500 meters per second to 4,500 meters per second [2]. An appropriate guess value for the wet mass is 150,650 kilograms.

Profit and Costs

The overall profit from the rocket is found using Equation (2).

$$Profit = R - C_f - C_d - C_e \tag{2}$$

Where C_f is the cost of fuel, given by:

$$C_f = 4.154(m_0 - m_f) \tag{3}$$

C_d is the cost of the rocket:

$$C_d = 154.36m_f \tag{4}$$

C_{ex} is the cost in relation to adjusting the exhaust velocity

$$C_{ex} = 75v_0 \tag{5}$$

R is revenue and is given by

$$R = 550\Delta v \tag{6}$$

References

[1] Holli Riebeek. Catalog of earth satellite orbits: Feature articles. 2009.

[2] Nesrin Sarigul-Klijn, Chris Noel, and Martinus Sarigul-Klijn. Air launching eart-to-orbit vehicles: Delta v gains from launch conditions and vehicle aerodynamics. 01 2004.