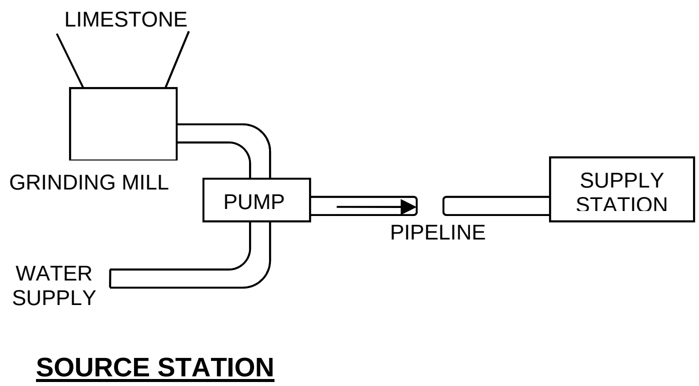
ME 575: Pipeline Design Problem

Design a pipeline for transporting crushed limestone from a quarry to a terminal located some distance away, using water as a transporting medium.



The limestone is crushed at the quarry, mixed with water to form a slurry, and pumped through the pipe. We would like to minimize the operating cost, which is primarily determined by the grinding power and the pumping power.

The following specifications are given,

- L = length of pipeline = 15 miles
- W = flowrate of limestone = 12.67 lb_m/sec
- a = average lump size of limestone before grinding = 0.01 ft.

The following need to be determined:

- V = average flow velocity, ft/sec
- c = volumetric concentration of slurry = $\dfrac{\text{vol. limestone}}{(\text{vol. water} + \text{vol. limestone})}$
- D = internal diameter of pipe, ft.
- d = average limestone particle size after grinding, ft.
- Q_w = flow rate of water, ft^3/sec
- ρ = density of slurry lb_m/ft^3

You should take into account:

The velocity, V, must exceed that at which sedimentation and clogging would occur. The formula for grinding power is not valid for a particle size below 0.0008 ft (particle size after grinding). The concentration of limestone in the pipe must be less than that at which pipe blockage would occur. The pipe diameter must not exceed 6 inches, above which the initial cost would be excessive.

The following expressions will be used to build a model:

Power for Grinding

$$P_g = 218\ W\left(\frac{1}{\sqrt{d}} - \frac{1}{\sqrt{a}}\right) \qquad (1)$$

where P_g has units of ft-lb_f/sec; W is in lb_m/sec and d, a are in ft. The constant 218 is a conversion factor that also has units; we will assume the units are such as to give P_g in ft-lb_f/sec.

Power for Pumping
The friction factor for the slurry is estimated by

$$f = f_w\left[\frac{\rho_w}{\rho} + 150c\,\frac{\rho_w}{\rho}\left(\frac{gD(S-1)}{V^2\sqrt{C_d}}\right)^{1.5}\right] \qquad (2)$$

where

f_w = friction factor of water
g = acceleration due to gravity = 32.17 ft/sec^2
ρ_w = density of water = 62.4 lb_m/ft^3
C_d = average drag coefficient of the particles
S = specific gravity of the limestone (density of limestone divided by the density of water)

The friction factor of water is given by,

$$f_{w1} = \frac{0.3164}{R_w^{0.25}} \qquad \text{if} \qquad R_w \le 10^5$$

$$f_{w2} = 0.0032 + 0.221 R_w^{-0.237} \qquad R_w \ge 10^5$$

Another option is to avoid the discontinuity altogether by finding another function that approximates both functions and has continuous first and second derivatives. One example of such function is to fit a higher order polynomial to f_w versus $\log_{10}(R_w)$ as shown below. This polynomial (or other function) would then be included in the model instead of f_{w1} and f_{w2}.

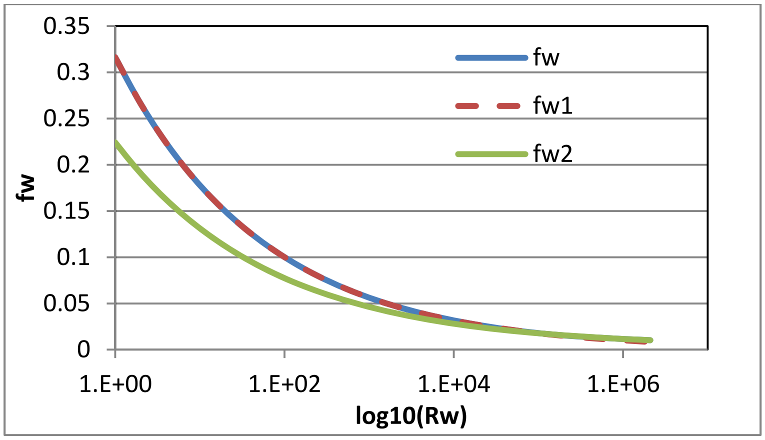


Fig 1. Plots of f_w, f_{w1}, and f_{w2} versus $\log_{10}(R_w)$

The factor R_w is defined as:

$$R_w = \frac{\rho_w VD}{\mu}$$

where

ρ_w = density of water
V = Velocity
D = diameter of pipe
μ = viscosity of water = 7.392×10^{-4} lb_m/(ft-sec) or 2.298×10^{-5} lb_f-sec/ft^2

Expression (2) above contains C_d, the drag coefficient of the particles. The drag coefficient combines with Reynold's number for the particle as a dimensionless quantity which depends on particle diameter:

$$C_d R_p^2 = 4g\rho_w d^3 \left(\frac{\gamma - \rho_w}{3\mu^2} \right)$$

where

γ = limestone density = 168.5 lb_m/ft^3
μ = viscosity of water
R_p = Reynolds number for the particle at terminal settling velocity.

There is an empirical relationship between C_d and $C_dR_p^2$ defined by the following table:

C_d	240	120	80	49.5	36.5	26.5	14.6	10.4
$C_d R_p^2$	2.4	4.8	7.2	12.4	17.9	26.5	58.4	93.7

C_d	6.9	5.3	4.1	2.55	2.0	1.5	1.27	1.07
$C_d R_p^2$	173	260	410	1020	1800	3750	6230	10,700

C_d	0.77	0.65	0.55	0.5	0.46	0.42	0.40	0.385
$C_d R_p^2$	30,800	58,500	138,000	245,000	460,000	1,680,000	3,600,000	9,600,000

The slurry density can be expressed as,

$$\rho = \rho_w + c(\gamma - \rho_w) \qquad \text{(units are } lb_m/ft^3\text{)} \qquad (3)$$

The pressure drop in the pipe due to friction is given by,

$$\Delta p = \frac{f \rho L V^2}{D 2 g_c} \qquad \text{(units are } lb_f/ft^2\text{)} \qquad (4)$$

where

- f = friction factor for the slurry, given by (2) above
- ρ = density of slurry, given by (3) above
- L = length of pipeline
- V = velocity of slurry
- D = diameter of pipe
- g_c = conversion between lb_f and lb_m = $32.17 \dfrac{lb_m\text{-}ft}{lb_f\text{-}sec^2}$

Finally, the friction power loss is given by

$$P_f = \Delta p\, Q \qquad \text{(units in ft-}lb_f\text{/sec)}$$

where

- Δp = is pressure drop from (4)
- Q = slurry flow rate (ft^3/sec)

Sedimentation

Sedimentation and clogging may occur if the velocity V is less than some critical velocity V_c. This velocity is estimated by the equation,

$$V_c = \left(\frac{40 g c (S-1) D}{\sqrt{C_d}} \right)^{0.5}$$

Pipe Blockage

Blockage can occur due to simply too high a fraction of solids in the slurry. If the particles were idealized into spheres of equal size and jammed together, the percent of unoccupied space, or voidage would be 26% or a concentration of 0.74. For irregular particles it is estimated that a safe concentration should be less than 0.4.

Miscellaneous
We will assume the velocity of the slurry, particles and water to all be the same.

The specification for flow rate of limestone is only tentative. **Examine how the optimal operating power changes if we change the flow rate of limestone.**

Comments
Note that this model requires a curve fit of some data to relate $C_d \operatorname{Re}^2$ to C_d. Make sure the curve fit is good enough that you do not introduce significant error in the problem (this implies the goodness of fit is high).

At the optimum the objective will be on the order of 300,000 ft-lb_f/sec.
Recall that mass flow rate is $\dot{m} = \rho AV$ and volumetric flow rate is $Q = AV$

This problem is taken from James Siddall, *Optimal Engineering Design*, Dekker.

Check your units!

For this assignment, you will develop the model from the description, code it, debug it, and optimize it. I will not be giving a sample design to check your model—you should devise a way to check on your own. You can work on this assignment in groups of two if you wish (only one report is required).

Report
Turn in an executive summary that gives the main optimization results. The summary may include a brief introduction to the problem and the model and any modeling equations that are not given in the assignment. In your summary, provide a brief description of the results of optimization including the optimum and the design space around the optimum. Do you feel this is a global optimum? You may want to include a contour plot to highlight the feasible region and your solution. You should decide what plots you should include. You need to demonstrate that you understand the meaning of the results you have included. Also, please provide a listing of your model in the Appendix.