Karush-Kuhn-Tucker Conditions
with Inequality and Equality Constraints

For a problem in the following form,

Min	$f(\mathbf{x})$		(1)
s.t.	$g_i(\mathbf{x}) - b_i \geq 0$	$i = 1, \ldots, k$	(2)
	$g_i(\mathbf{x}) - b_i = 0$	$i = k+1, \ldots, m$	(3)

A) Give below the KKT necessary conditions, explaining each equation.

Description	Equation	Applies to
Feasibility		
No direction which improves objective and is feasible		
Complementary slackness		
Positive Lagrange multipliers		

B) Solve for the optimum using the KT conditions

Min $f = 4x_1^2 + 2x_2^2$

s.t. $3x_1 + x_2 = 8$

$2x_1 + 4x_2 \leq 15$

Note: at the optimum, it is known that the inequality constraint is satisfied but not binding. Take advantage of this information.

C) For the following problem,

Min $f = x_1^2 + x_2$

s.t. $g_1 = x_1^2 + x_2^2 - 9 \le 0$

$g_2 = x_1 + x_2 - 1 \le 0$

Show that the point [1,0] does not satisfy the KT conditions