K-T Equations: Inequalities

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Solving K-T Equations with Inequalities

Solve for the optimum to the following problem using the KT conditions,

Min $f = x_1^2 + 2x_2^2 + 3x_3^2$

s.t. $g_1 = -5x_1 + x_2 + 3x_3 \le -3$

$g_2 = 2x_1 + x_2 + 2x_3 \ge 6$

Example continued

Step 1: Change problem to be in the proper form

Min $f = x_1^2 + 2x_2^2 + 3x_3^2$

s.t. $g_1 = 5x_1 - x_2 - 3x_3 \geq 3$

$g_2 = 2x_1 + x_2 + 2x_3 \geq 6$

Step 2: Assume both constraints are binding (we will not know if this is correct until we see the signs of the Lagrange multipliers)

Example continued

Step 3: Write out the Lagrange multiplier equations:

$$\frac{\partial f}{\partial x_1} - \lambda_1 \frac{\partial g_1}{\partial x_1} - \lambda_2 \frac{\partial g_2}{\partial x_1} = 0$$

$$\frac{\partial f}{\partial x_2} - \lambda_1 \frac{\partial g_1}{\partial x_2} - \lambda_2 \frac{\partial g_2}{\partial x_2} = 0$$

$$\frac{\partial f}{\partial x_3} - \lambda_1 \frac{\partial g_1}{\partial x_3} - \lambda_2 \frac{\partial g_2}{\partial x_3} = 0$$

$$g_1 = 3$$

$$g_2 = 6$$

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Example continued

$$2x_1 - \lambda_1(5) - \lambda_2(2) = 0$$

$$4x_2 - \lambda_1(-1) - \lambda_2(1) = 0$$

$$6x_3 - \lambda_1(-3) - \lambda_2(2) = 0$$

$$5x_1 - x_2 - 3x_3 = 3$$

$$2x_1 + x_2 + 2x_3 = 6$$

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Solution

$$x_1 = 1.450$$

$$x_2 = 0.800$$

$$x_3 = 1.150$$

$$\lambda_1 = -0.50$$

$$\lambda_2 = 2.70$$

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What do we do?

- Drop the constraint
- Solve the problem again

$$x_1 = 2.057$$

$$x_2 = 0.5143$$

$$x_3 = 0.6857$$

$$\lambda_1 = 0.$$

$$\lambda_2 = 2.057$$

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Contour Plot