Karush-Kuhn-Tucker Conditions for Equality Constraints

For a problem in the following form,

$$\text{Min} \quad f(\mathbf{x}) \qquad (1)$$
$$\text{s.t.} \quad g_i(\mathbf{x}) - b_i \geq 0 \quad i = 1, \ldots, k \qquad (2)$$
$$g_i(\mathbf{x}) - b_i = 0 \quad i = k+1, \ldots, m \qquad (3)$$

A) Give below the KKT necessary conditions, explaining each equation.

Description	Equation	Applies to
Feasibility		
No direction which improves objective and is feasible		
Complementary slackness		
Positive Lagrange multipliers		

B) Given the following problem, solve for the solution using the KKT Conditions.

Min $\quad f = 2x_1^2 + x_2^2 + 4x_3^2$

s.t. $\quad \begin{aligned} g_1 &= x_1 + 2x_2 - x_3 = 6 \\ g_2 &= 2x_1 - 2x_2 + 3x_3 = 12 \end{aligned}$

Hint:

$$A := \begin{pmatrix} 1 & 2 & -1 & 0 & 0 \\ 2 & -2 & 3 & 0 & 0 \\ 4 & 0 & 0 & -1 & -2 \\ 0 & 2 & 0 & -2 & 2 \\ 0 & 0 & 8 & 1 & -3 \end{pmatrix} \qquad b := \begin{pmatrix} 6 \\ 12 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad A \cdot x = b \qquad x := A^{-1} \cdot b \qquad x = \begin{pmatrix} 5.045 \\ 1.194 \\ 1.433 \\ 7.522 \\ 6.328 \end{pmatrix}$$