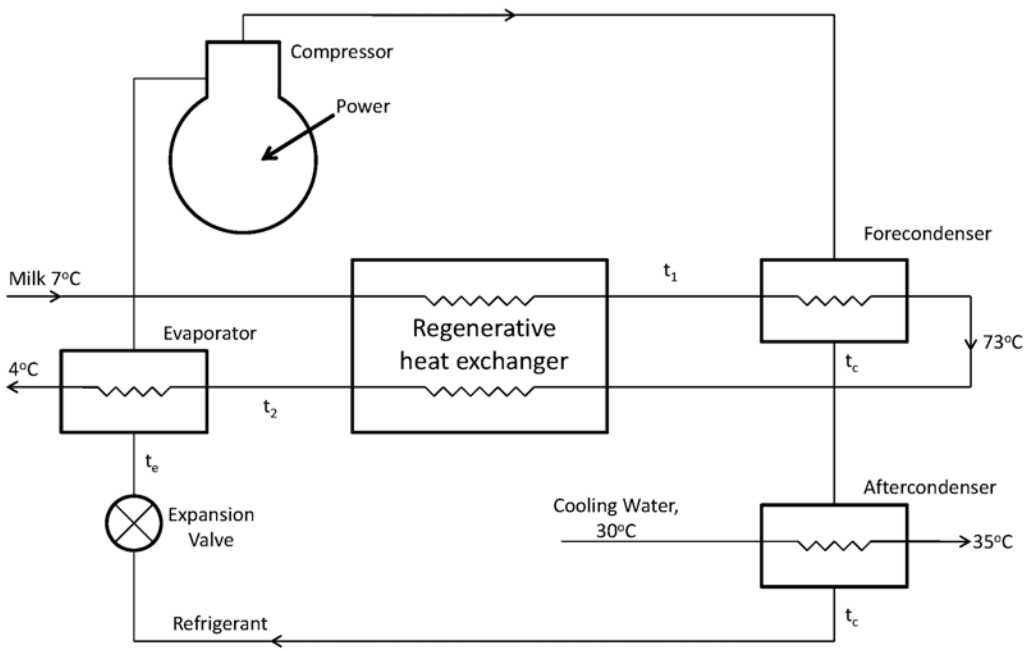
ME 575: Heat Pump for Pasteurizing Milk

In the pasteurization of milk the temperature is raised to 73°C, held for 20 sec., and then cooled. The milk arrives at a temperature of 7°C and is delivered from the pasteurizing process for packaging at a temperature of 4°C.

We will consider using a heat pump with a regenerative heat exchanger to do this. One possible cycle is shown in the figure below. The incoming milk is preheated in a regenerative heat exchanger and then heated further in the fore-condenser of the heat pump. As it exits the fore-condenser, the temperature of the milk is 73°C. Thereafter the milk is cooled as it flows through the other side of the regenerative heat exchanger and then through the evaporator of the heat pump.



Optimization Problem

Find the optimal heat pump and regenerative exchanger that minimizes the total present worth of costs (capital cost and operating cost). Specifically, determine the areas of the heat exchangers (evaporator, condensers, and regenerative exchanger), size of the compressor, and temperatures t_1, t_2, t_e and t_c that result in the minimum total cost. Run the problem with no constraints on temperatures of approach (Case 1); then rerun with constraints that all temperatures of approach (evaporator, condensers, regenerative heat exchanger) are at least 10 °C (Case 2).

Demonstrate good practice in developing the model, setting up the design space, optimizing the model and interpreting the results, as we have discussed in class and in the notes.

Modeling

The problem is modeled using principles and equations from thermodynamics and heat transfer. A heat pump is a device that "pumps" heat from a colder space to a hotter one. This requires work; however one unit of work energy can potentially pump several units of heat energy. This ratio is called the "coefficient of performance" (COP). For this heat pump COP is defined by,

$$COP = \frac{Q_e}{W_{comp}} \qquad (1)$$

where,

Q_e = heat transferred at the evaporator (W)

W_{comp} = energy to the compressor (W)

The most efficient heat pump possible is a Carnot heat pump. It can be shown that for a Carnot heat pump Eq (1) can also be written,

$$COP = \frac{t_e}{\left(t_c - t_e\right)} \qquad (2)$$

where,

t_e = evaporating temperature <u>in absolute units</u> (°K)

t_c = condensing temperature <u>in absolute units</u> (°K)

We will assume our heat pump has a COP which is 75% of the Carnot COP for the same evaporator and condensing temperatures.

If we put a control volume around the entire system and look at where energy crosses the boundaries, we have work coming in to the compressor, milk coming in at 7 °C and leaving at 4 °C, and water for the after-condenser coming in at 30 °C and leaving at 35 °C. At steady state the first law requires,

$$W_{comp} + \dot{m}_{milk}\left(h_{milk,in} - h_{milk,out}\right) = \dot{m}_{water}\left(h_{water,out} - h_{water,in}\right) \qquad (3)$$

where the first and second terms can be computed by,

$$\dot{m}_{milk}\left(h_{milk,in} - h_{milk,out}\right) = \dot{m}_{milk} C_{p,milk}\left(t_{milk,in} - t_{milk,out}\right) \qquad (4)$$

$$\dot{m}_{water}\left(h_{water,out} - h_{water,in}\right) = \dot{m}_{water} C_{p,water}\left(t_{water,out} - t_{water,in}\right) = Q_{ac} \qquad (5)$$

and,

$\dot{m}_{milk}$ = mass flow rate of milk (kg/s)

$C_{p,milk}$ = specific heat for milk (kJ/(kg · C)

with similar notation for water

Q_{ac} =energy transferred in the after-condenser (W)

If we substitute Eqs. (4) and (5) into (3) we have,

$$W_{comp} + \dot{m}_{milk} C_{p,m} \left(t_{milk,in} - t_{milk,out} \right) = Q_{ac} \qquad (6)$$

The heat transferred in the evaporator, condensers and regenerative exchanger can be modeled by,

$$Q = UA\Delta T$$

where

Q = heat transferred, (W)
U = overall heat transfer coefficient, (W/(m² · °C))
A = area, (m²)
ΔT = log mean temperature difference, (°C)

The log mean temperature difference (in terms of generic temperatures) is given by,

$$\Delta T = \frac{(t_{h2} - t_{c2}) - (t_{h1} - t_{c1})}{\ln\left[(t_{h2} - t_{c2)} /(t_{h1} - t_{c1})\right]}$$

where, $\left(t_{h2} - t_{c2}\right)$ is, for example, the difference in the temperature of the hot and cold fluids at one end of the exchanger, denoted by subscript 2. This difference is called the “temperature of approach.” The temperature of approach on the other side is $\left(t_{h1} - t_{c1}\right)$.

Finally, the heat transferred in the evaporator, condensers and regenerative heat exchanger must equal the sensible heat gain/loss of the milk or water. For example, at the evaporator,

$$Q_e = Q_{e,milk} = \dot{m}_{milk} C_{p,milk} \left(t_2 - 4 \right) \qquad (7)$$

where

t_2 = milk temperature entering evaporator as given in diagram, (°C)
4 = milk temperature exiting evaporator as given in diagram, (°C)

Similar equations to Eq. (7) apply for the other heat exchangers in the problem. However, you cannot write such equations for the refrigerant side of the evaporator and condensers.

Data

Flow rate of milk, 4 kg/s for 4h/day
Specific heat of milk, 3.75 kJ/(kg · °C)
Cost of all heat-exchanger surfaces (evaporator, condensers, and regenerative heat exchanger), $200 per square meter
U value of regenerative heat exchanger, 500 W/(m² · °C)
U values for evaporator, fore-condenser, and after-condenser, 600 W/(m² · °C)
Cost of the compressor, $240 per kilowatt of power

Cost of electricity, 7 cents per kilowatt hour
Interest rate, 9 percent
Economic life of the plant, 6 years
Cooling water enters the after-condenser at 30°C and leaves at 35°C

Sample Design

A sample design for this problem is not given; however, you can estimate a design by choosing reasonable values for temperatures and from these calculating areas, energy consumed and cost.

Report

For this problem, provide an executive summary of main optimization results (include values for design variables and objective and indicate which constraints are binding) for the two cases. Include analysis to show that the solution is at an optimal solution. Some of things that you may want to include are:

1) A contour plot (Case 2) of the design space showing the objective and constraints as a function of t_e, t_c, with the feasible region shaded and optimum marked.
2) Discuss any observations or comments about the model, process of optimization or the design space. Do you feel this is a global optimum?
3) Appendix: listing of your model file

Comments

The math function for the natural log is "`log()`" in both OptdesX and APMonitor

Without using MPECs (more advanced topic), there is no way to include conditional statements in APMonitor that can be handled by solvers such as APOPT and IPOPT. The optimal solution will require that you use the average temperature difference instead of the log-mean temperature difference (LMTD) for the regenerative heat exchanger. All of the other heat exchangers can use the LMTD. You can also put a small constraint to avoid the divide by zero for the LMTD.

You may wish to study the heat pump example problem given in Chapter 2, Section 9 of the notes.

This problem is based on a problem from W. Stoecker, *Design of Thermal Systems*, 3rd ed. You may work on this problem in pairs if you wish; only one report is necessary.