Introduction to Discrete Optimization

Optimization Techniques in Engineering

John Hedengren

Discrete Variables in Engineering Design

- Continuous: Variables that can attain any value between a lower and upper bound
- Discrete: Variables that are only feasible at particular values
  - Binary: $x \in \{0, 1\}$ or $x \in \{False, True\}$
  - Integer: $x \in \{2, 3, 4, 5\}$ or $x \in \{0, 2, 5, 10\}$
  - Discrete: $x \in \{0.5, 1.0, 2.0\}$
  - Sets: $x \in \{London, New\ York, Boston, Tokyo\}$
- Examples:
  - Motor size, pump size, valves, fasteners, pipes, etc.

Addition of Discrete Variables

Continuous

$$\min f(x)$$
$$s.t.\ 0 = g(x)$$
$$0 < h(x)$$
$$x \in \Re^n$$

Discrete

$$\min J(x,z)$$
$$s.t.\ 0 = g(x,z)$$
$$0 < h(x,z)$$
$$x \in \Re^n \ z \in \mathrm{I}^m$$

Exhaustive Search

- Evaluate all of the possible solutions
- Example 1:
  - A sprinkler system can have 1 of 5 sizes of pipe, 1 of 3 sizes of pump, and 1 of 10 types of sprinkler heads. How many combinations are possible?
    - $5\ x\ 3\ x\ 10 = 150\ Possible\ Combinations$
- Example 2:
  - There are 10 standard sprinkler heads that could be chosen to optimally distribute water in a golf course. If we can choose 25 sprinkler heads in the design, how many combinations are possible?
    - $10^{25} \approx Number\ of\ Grains\ of\ Sand\ on\ Earth$

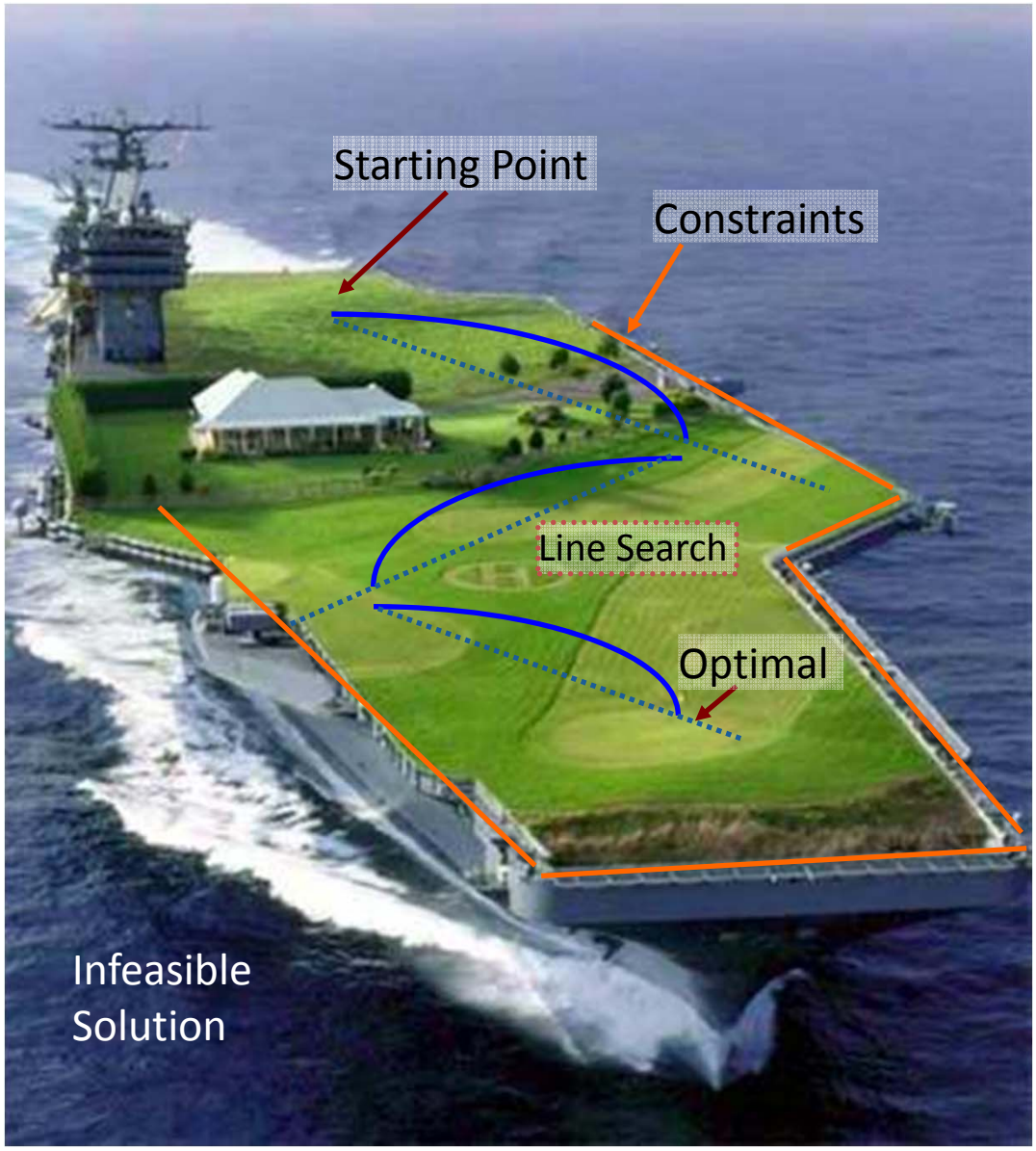
Sampling vs. Gradient Techniques

- Sampling-Based Techniques
  - Trail Values -> Objective Function -> New Trial Values
  - Do not require 1st or 2nd derivative information
  - Examples:
    - Exhaustive Search
    - Simulated Annealing
    - Genetic Algorithms
- Gradient-Based Techniques
  - Faster when gradients are available (~ 1000x faster)
  - Examples: Branch and Bound, Outer Approximation

Topics Covered this Week

- Exhaustive Search
- Branch and Bound
- Simulated Annealing
- Genetic Algorithms

Continuous Optimization



Continuous Optimization

Objective: Find lowest elevation on the golf course

Golf Course is Typically Convex or Non-Convex?

Exhaustive Search

2 Decisions (white and black)

Possible solutions at intersection of grid locations

Place a golf ball at each grid location and evaluate best

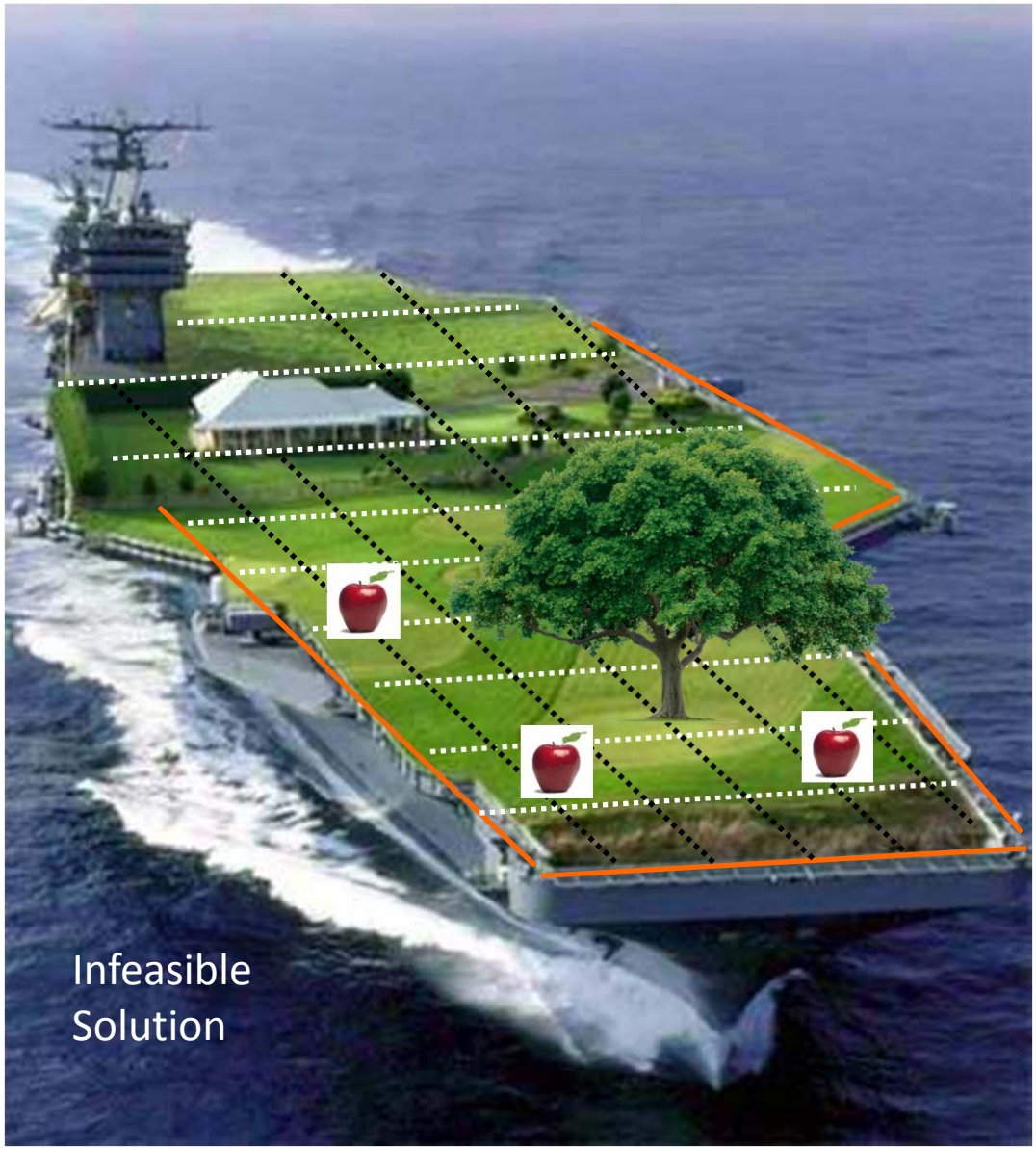
Simulated Annealing

Drop bouncing balls from helicopter at some grid locations and evaluate best once the balls have stopped bouncing

Genetic Algorithm

Place gerbils at some grid locations. Eliminate higher elevation gerbils with a hawk and evaluate lowest gerbil elevation once the population has gone through many generations.

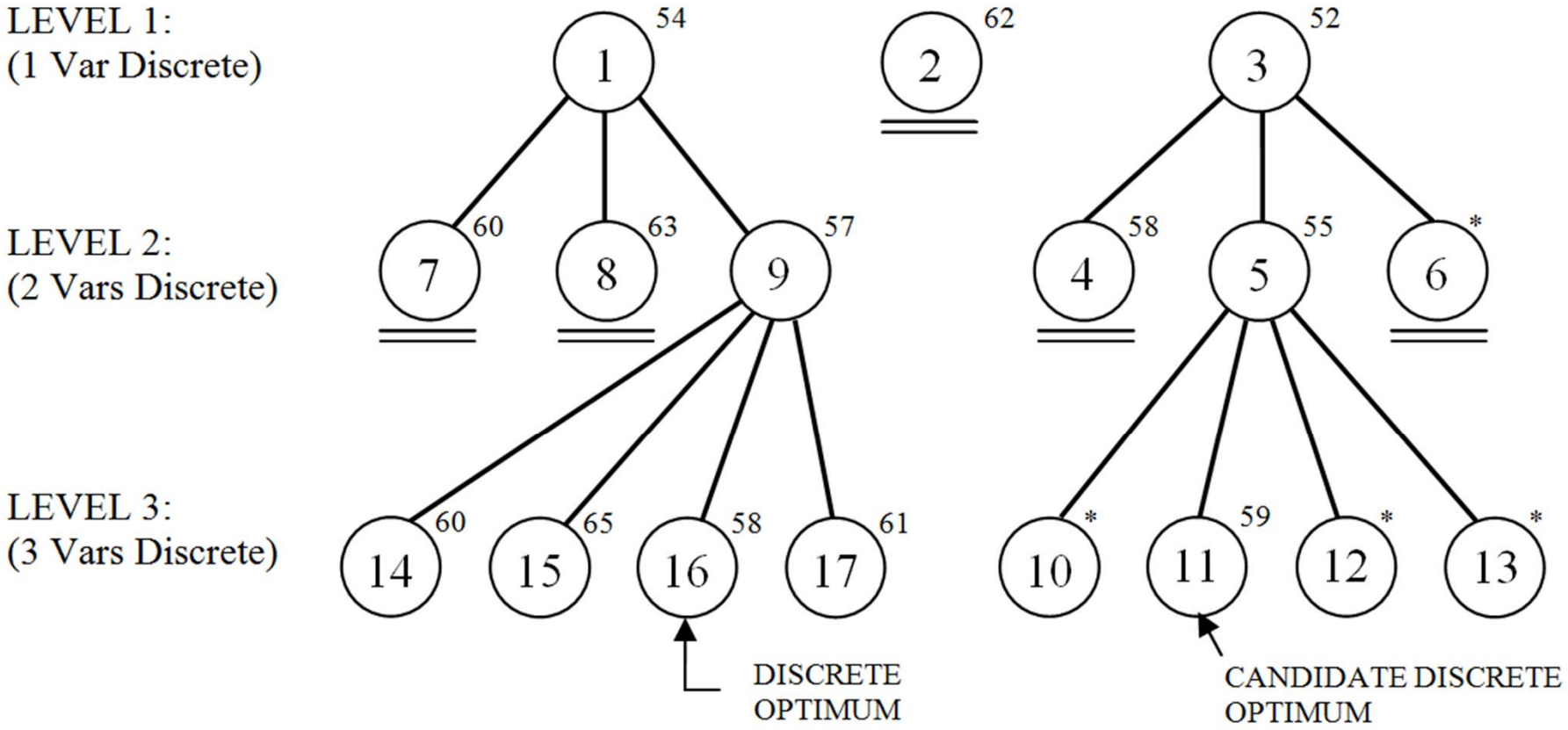
Branch and Bound



Plant an apple seed at the optimal relaxed solution (with all variables continuous). Determine the elevation of the apples that drop and cut off branches that produce higher elevation apples than the best integer solution.

SCHEMATIC OF BRANCH AND BOUND METHOD

* INFEASIBLE ═══ PRUNED



Example Problem

- Branch and Bound Activity
  - MATLAB Solution
  - APOPT Solution