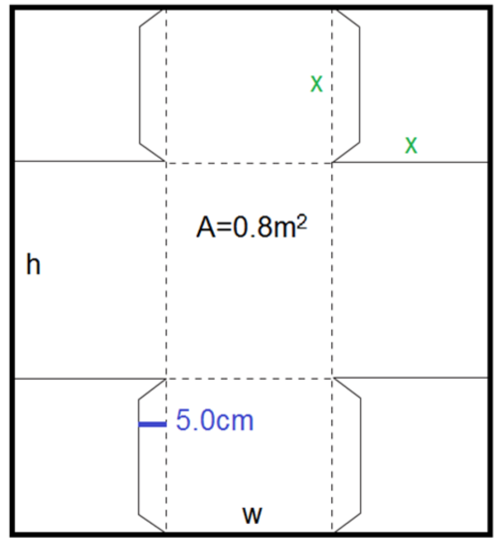
Optimization: Maximize the Volume of a Box

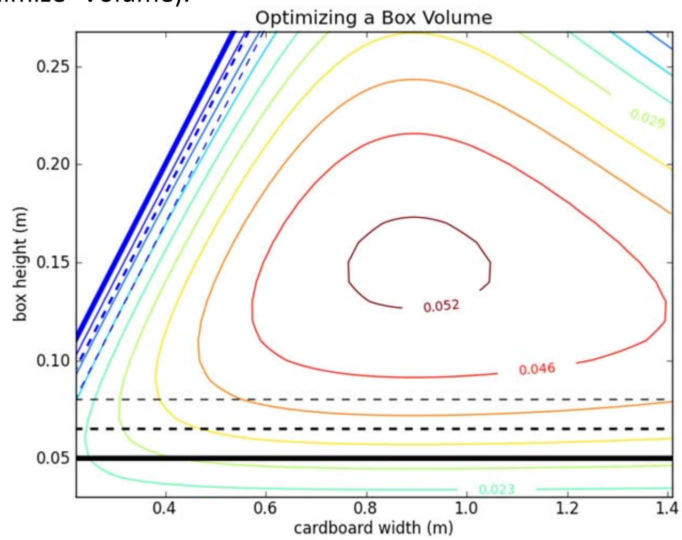
A piece of cardboard with a total area of 0.8m^2 is to be made into an open-top box by first removing the corners and then by folding the box sides up and securing the tabs to the adjacent box side. The starting cardboard sheet has height ***h*** and width ***w***. When cut and folded, the box has a width of ***w-2x***, a length of ***h-2x***, and a height of ***x***. In order to properly secure the tabs to the adjacent box side, the width of the tab must be 5 centimeters (0.05m). The objective is to **maximize the volume of the box** by choosing an appropriate value of ***x*** (the height of the box) and ***w*** (the starting width of the cardboard sheet).



1. Develop an expression for the volume of the box as a function of ***x*** and ***w*** only. *Hint:* The height ***h*** and width ***w*** are related by the total area.

2. Determine the optimal volume of the box. Differentiate the objective with respect to ***x*** and ***w*** and set each equation equal to zero. Solve the resulting two equations for optimal values of ***x*** and ***w***. Remember that for $ax^2 + bx + c = 0$ the solution is the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

3. Show the first iteration of the steepest descent method starting from w=1.0m and x=0.1m and alpha=0.2. Do not normalize the search vector. Plot the starting and first iteration point on the contour plot. Remember that (maximize Volume) = (minimize -Volume).



4. What influence do the side tab constraints have on the optimal solution?