Application Project

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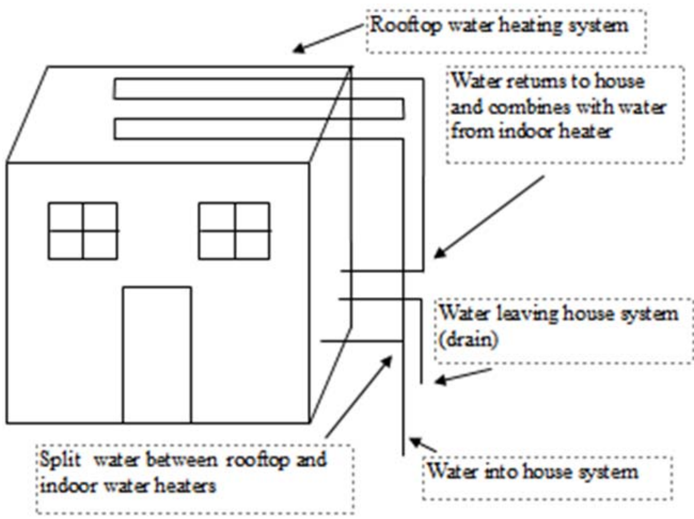
Problem Write-up

You are a homeowner trying to save money on your water heating costs. You foresee the possibility that fracking may be outlawed that and the price of natural gas may rise dramatically. Rather than relying on gas only for your water heating, you want to build a simple solar heating system. You will divert water from your heater to a system of pipes on your roof. While the sun shines, the pipes on the roof will be heated (heating your water). This water will flow back into the house, rejoin any additional hot water that needs to be provided by the gas heater. You want your system to be able to assist with water heating for about one hour of water flow every day.



The problem is that you need to decide how much pipe to buy and install on your roof. You need to balance the cost of the pipes over their lifetime against the savings from your gas heater. More piping on the roof gives more surface area which means more sunlight absorbed. A larger pipe system also means a larger material cost. To increase surface area, you can buy pipe of various diameters, thicknesses, and lengths. To maximize your savings from buying gas for heating, you can also vary what fraction of the incoming water is diverted from the gas heater to the rooftop heater.

In addition to maximizing your savings, there will be other factors to consider. If your pipe system is too long, you will lose too much water pressure from friction losses. The pipe thickness will have to increase as the diameter of the pipe increases to ensure adequate mechanical strength. The inside diameter of the pipe can't exceed 6 cm, or else cost dramatically increases (researched prices increased ~4 times after this diameter). Pipe is only available with an inside diameter as small as 6 mm. At least ½ m of pipe will be used on the roof but no more than 30 m. Pipe is available with thicknesses varying from 1 mm to 1.5 cm. With these thickness and diameter restrictions we are able to assume that the temperature of the pipe and the water are effectively equal (i.e. ignore radial conduction through the pipe). At least 10% of the water will be diverted to the roof, but no more than 90%. Water can only be heated up to 100 degrees Celsius.

Table 1 shows the known values for your area and your house:

Table 1--Important values

Labor cost for installation	$	500
Cost of pipe per volume of material	$/(m^3)	40000
Predicted Cost of natural gas heating	$/kW-hr	0.5
Reflectivity of pipe (painted black)		0.01
Inlet Pressure	Pa	482633
Inlet water Temperature	C	20
Convection Coefficient	W/K/m^2	5
Average Air Temp	C	20
Temp after all heating	C	60
Total Water flow rate	m^3/s	0.0001
Height of roof from water inlet	m	4
Friction factor		0.1
Minimum delivered pressure	Pa	344633
Flux from the sun	W/m^2	300
Lifetime of pipe system	yrs	5

Important equations for your design and optimization will include the following:

$$Q_{free\_convection} = h_{air} \cdot Area_{top\_half} \cdot \left(Temp_{pipe\_average} - Temp_{Air}\right)$$

$$Q_{heater} = mass\_flow_{water} \cdot C_p \cdot \left(Temp_{heater\_out} - Temp_{in}\right)$$

$$Q_{sun} = Flux_{sun} \cdot Area_{covered} \cdot (1 - reflectivity)$$

$$Pressure_{loss\_max} = \rho_{water} \cdot gravity \cdot height_{roof} + \frac{\left(length_{pipe} + 2 \cdot height_{roof}\right)}{diameter_{inside\_pipe}} \cdot \rho_{water} \cdot \frac{{v_{water}}^2}{2}$$

Note that the relevant area for convective heat transfer from the pipe is the top half of the pipe on the roof. The bottom half of the pipe (which doesn't see the sun) will be insulated, as will the pipe between the roof and the ground. The relevant area for absorbing heat from the sun is the "covered" area, the rectangular area covered by the pipe across the roof.