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ME 575

Project 1
Turbomachinery Efficiency Optimization

Introduction

Jet engines are utilized in an increasing number of applications, from power generation to propulsion. One of the biggest concerns in the design of a jet engine is the efficiency of the compressor. The compressor adds work to an airflow, increasing both its pressure and temperature. The air enters the combustor after the compressor, which further increases the temperature. The heated air then passes through a turbine which extracts enough energy to power the compressor, and the remainder of the energy in the air is transformed into thrust via a nozzle (see Figure 1). The efficiency of a compressor determines how much energy is left after the turbine to become thrust, and so very strongly affects the maximum thrust that a jet engine can produce.

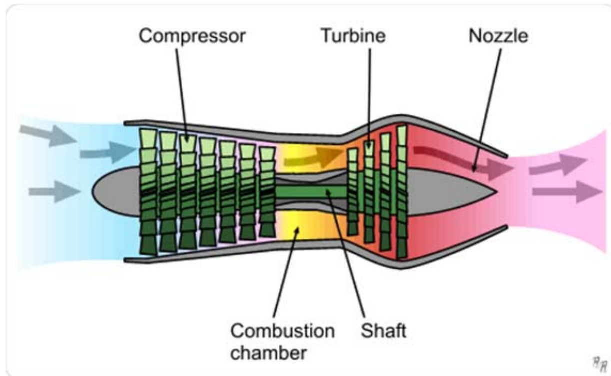


Figure 1-Cross Section of a Turbojet

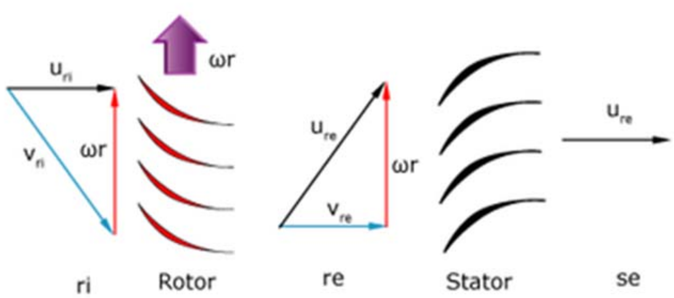


Figure 2-Rotor/Stator Blade Cascade

The efficiency of a compressor is determined by the shape of the blades that make up the compressor and the relation of the different rows of blades to each other. Each row of rotors is turning very quickly, which causes the air to turn as it passes through each. After each rotor row is a row of stators whose main purpose is to straighten the flow so that the next row of rotors can work as efficiently as possible (see Figure 2). Each row of blades can be rotated to provide a better angle to the flow and a better efficiency.

Theory

While true compressor optimization takes years of training and a large amount of time, a basic problem can be formulated which simplifies the governing principles. For this problem, we assume that every blade in the compressor has a constant cross-section. We will use the NACA 4212 airfoil as the specified cross section. Each blade row will have the same number of blades, and will be similar in every respect except for the direction they are turned in. The formulation for this problem follows.

$$\eta = \phi\left(\left(\frac{\%R - \phi\epsilon_r}{\phi + \epsilon_r\%R}\right) + \left(\frac{1 - \%R + \phi\epsilon_s}{\phi - \epsilon_s(1 - \%R)}\right)\right)$$

This is the governing equation for efficiency. The flow coefficient (ϕ) is defined as

$$\phi = \frac{c_a}{U}$$

Where C_a is the axial velocity through the compressor ($c_a = \frac{\dot{m}}{\rho_1 A_1}$) and U is the rotor velocity ($U = R\omega$) with R as the mean radius of the blade row and ω as the rotational speed of the blade row. ϵ_r and ϵ_s are characteristics of individual blades (r for the rotor, s for the stator), defined as

$$\tan(\epsilon_r) = \frac{C_{Dr}}{C_{Lr}}$$

$$\tan(\epsilon_s) = -\frac{C_{Ds}}{C_{Ls}}$$

C_D and C_L are the coefficients of drag and lift. Calculating the coefficients of drag and lift requires very advanced math and a highly detailed understanding of the geometry of the airfoil, and so they are generally found experimentally, as some relation to the angle of attack of the blade (AA). Values for the coefficients of lift and drag are found in Figures 3 and 4. (You will have to extract data points and fit a curve to them in order to make an equation and calculate these values.)

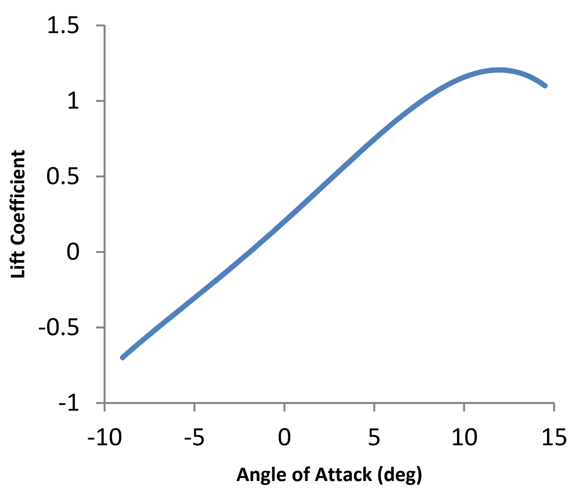


Figure 3-Coefficient of Lift vs. Angle of Attack

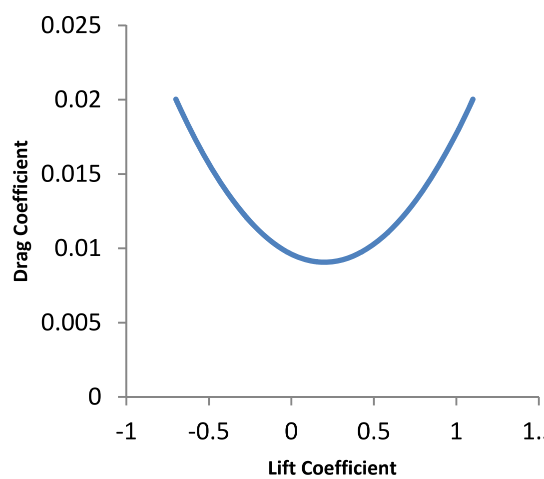
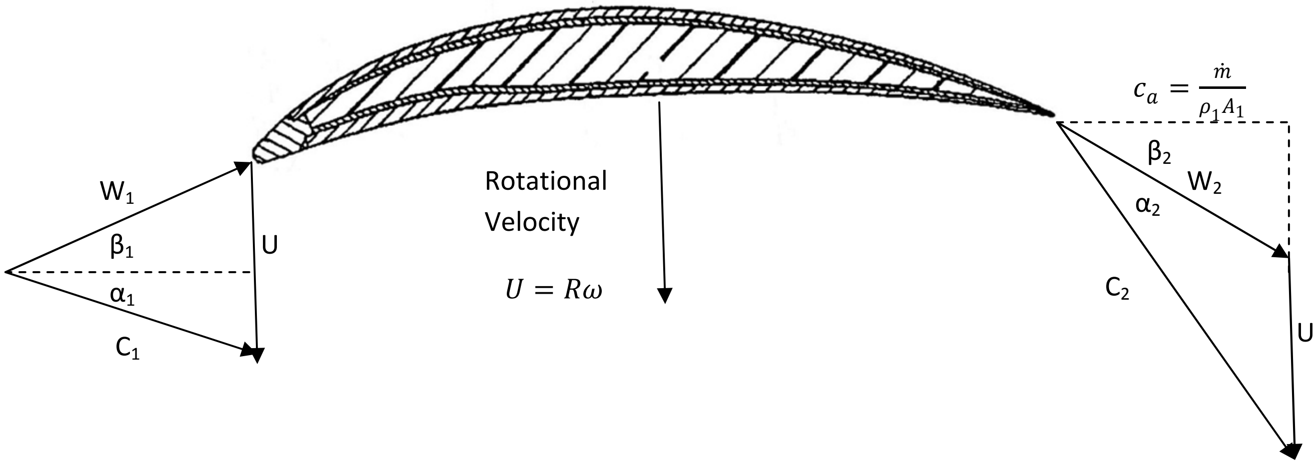


Figure 4-Coefficient of Drag vs. Coefficient of Lift

Percent Reaction (%R) is defined as

$$\%R = \frac{1}{1 + \left(\frac{c_2^2 - c_1^2}{w_1^2 - w_2^2}\right)}$$

where C_2, C_1, W_2, and W_1 are components of a velocity triangle. These values are calculated mainly using trigonometry, and a brief explanation is included here.



Air generally enters a compressor at some angle α_1 and velocity C_1. However, because of the rotational velocity of the rotor blades, U, the blade actually sees some other angle β_1 and velocity W_1. The axial component of both of these velocities is the overall axial velocity of the compressor (which is constant due to conservation of mass, since we have no change in area for the blade rows in this problem). As long as α_1, U, and the axial velocity can be found, the rest of the triangle can be created through simple trigonometry. We assume that the axial velocity stays constant through the compressor, and the rotational speed will of course stay constant across a single blade. With those two quantities known, the only other component needed is one of the angles of the triangle. This can be found with the turning angle of the rotor blade δ.

$$\beta_2 = \beta_1 + \delta_r$$

assuming that β_1 has properly been defined as a negative value. The development of the equations to find the velocity triangles for the stator is very similar. The absolute velocity and angle the flow leaving the rotor defines the absolute angle and velocity of the flow entering the stator. Again, the axial velocity has to remain constant in order to conserve mass through the compressor. The relation between the entry and exit angle for the stator is defined as

$$\alpha_2 = \alpha_1 - \delta_s$$

Both values of α should be positive in this relation. Once the velocity and angle at the stator exit are calculated, these become the inputs for the next row of rotor blades, which continues through as many blade rows as the compressor contains.

One point to be cautious of is that the rotor blades should not break the sound barrier, as this puts them under undue stress and can cause premature failure of the compressor. The speed of sound is defined as $a = \sqrt{\gamma * R * T_{in}}$ where R is the gas constant of air and γ is the ratio of specific heats in air

You will optimize the efficiency of a 3 stage compressor using the following parameters:

P_{in} = 272 psia	δ_r = 19°	Subject To:
T_{in} = 1310 R	δ_s = 22°	200 lbm/s ≤ $\dot{m}$ ≤ 500 lbm/s
$R_{mean,\ blade\ row}$ = 13.2 in.	AAR1 = 0°	5000 rpm ≤ ω ≤ 15000 rpm
$T_{blade\ row}$ = 1.24 in.	γ = 1.4	0° ≤ AA (all) ≤ 15°
α_1 = 15°	R = 1716 ft*lbf/slug*R	W1(rotor 1) < a