ME 575: Ideal Turbofan with fan exhausted design

Mid-Term Exam

Jet engines are used throughout the aerospace industry to generate substantial amounts of power, in order to power the flight of large aircraft. All jet engines generally intake air, compress the intake, ignite the compressed air, recapture energy with a turbine, and expel the remaining hot gases producing thrust. In order to perform this operation, jet engines comprise of an intake diffuser, a compressor, a burner stage, a turbine, and exit nozzle. There are several types of jet engines, varying in complexity of parts and number of sections to change the overall performance characteristics. A turbofan is a type of jet engine which offers a comparatively significant thrust force, while maintaining a good fuel economy. Turbofans are frequently used on commercial airliners, because of the requirement to carry large amounts of cargo in a cost effective manner. Recently, Boeing has been using the Rolls-Royce Trent 1000 and General Electric GEnx, two types of turbofans with fan exhausted on the Boeing 787. Because the components of a jet engine are interconnected, the selection of jet engine components can have a significant impact on total performance requiring an optimization approach.

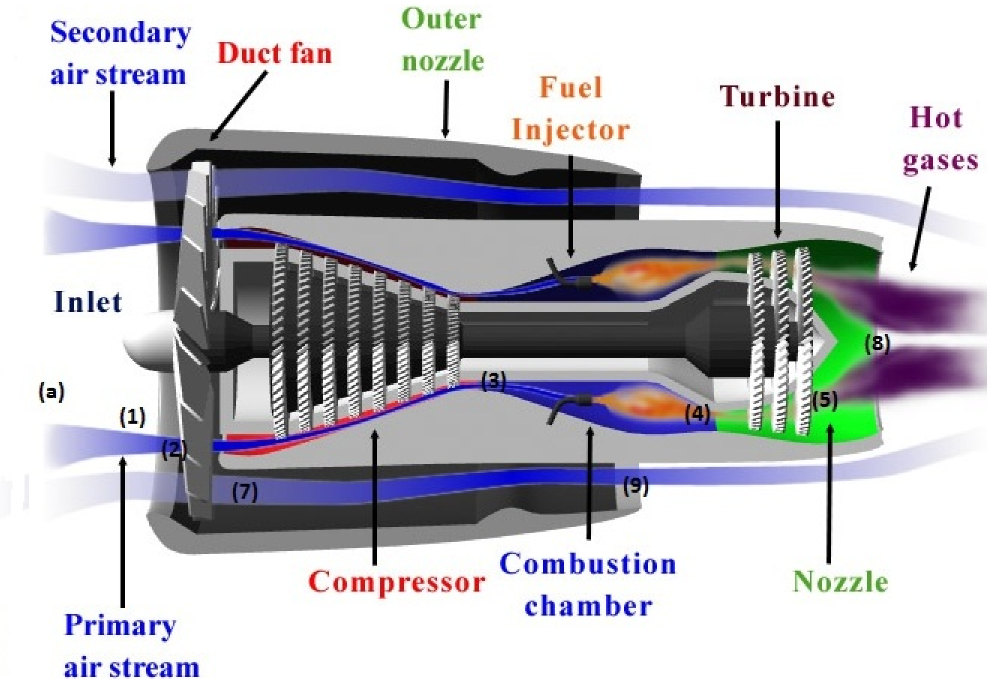

Figure 1: A Labeled Cross-Sectional View of a Turbofan

We will consider using a turbofan with fan exhausted in the ideal case to find the optimum selection of components for the new Boeing 787 to maximize thrust, thus increasing the overall allowable luggage weight of the airliner. The new 787 is designed to use jet engines with a maximum inlet area of 12 ft^2. In addition to the description above, a turbofan with fan exhausted includes a duct fan and outer nozzle (referred as the fan nozzle) as annotated on Figure 1. As a result of previous contracts, the burner, turbine, and nozzles have already been selected and allow a maximum mass flow rate of 250 (lbm/s). The feasible fans and compressors available have a fan and compressor pressure ratio (π_f and π_c) limitations of 1 to 5 and 5 to 45, respectively. While the bypass duct (Bypass Ratio, α) may also vary in size to optimize the compressor and fan performace with a maximum limitation of 5. Because this airliner will be flying the majority of its life at cruising speed, the optimum performance should be determined for the jet engine operating at cruising speed. This ideal turbofan will be flying at sea level ($T_a = 577.1°R$ and $P_a = 14.69 psi$) at a Mach number of 0.75 ($M_a = 0.75$).

The typical method in calculating overall performance for a jet engine requires stepping through each component beginning at the inlet. In order to clarify the calculations for each step, a standard numerical labeling system has been applied to the model, as seen in Figure 1. Below is a list of the variables and equations which influence jet engine performance.

Constants

Gravity, (ft/s^2), g = 32.17
Ratio of specific heats, γ =1.40
Density of air, $(slug/ft^3)$, ρ = 0.002376
Specific Heat (lbm/s), C_p = 0.24
Ideal Gas Constant (Btu/lbm-R), R = 53.35

Variables

Mass flow rate $\dot{m}$ (lbm/s)
Bypass ratio, α
Speed of sound, A_a (ft/s)
Speed of aircraft, U_a (ft/s)
Total atmospheric temperature
Total atmospheric pressure
Diffuser inlet area, A_{in} (ft^3)
Diffuser outlet total pressure, P_{t2} (psi)
Diffuser outlet total temperature, T_{t2} (R)
Fan pressure ratio, π_f
Fan outlet total pressure, P_{t7} (psi)
Fan outlet total temperature, T_{t7} (R)
Fan outlet atmospheric pressure, P_9 (psi)
Fan nozzle total pressure, P_{t9} (psi)
Fan nozzle total temperature, T_{t7} (R)
Fan outlet total Mach number, M_9
Fan exit static temperature, T_9 (R)
Speed of sound at fan exit, A_9 (ft/s)
Fan exit velocity, U_9 (ft/s)
Compressor pressure ratio, π_c
Compressor out total pressure, P_{t3} (psi)
Compressor out total temperature, T_{t3} (R)
Burner total pressure, P_{t4} (psi)
Fuel flow , $\dot{m}_f$ (lbm/s)
Turbine outlet total temperature , T_{t5} (R)
Turbine outlet total pressure , P_{t5} (psi)
Nozzle total pressure, P_{t8} (psi)
Nozzle total temperature, T_{t8} (R)
Exit pressure, P_8 (psi)
Mach number at nozzle exit, M_8
Exit temperature, T_8 (R)
Speed of sound at nozzle exit, A_8 (ft/s)
Speed of air at exit, U_8 (ft/s)
Thrust, $Thrust$ (lbf)
Thrust specific fuel consumption, $TSFC$ $(lbm/(h-lbf))$

Equations

Atmospheric Conditions

$A_a = \sqrt{\gamma R g T_a}$	Calculate Speed of Sound at Atmospheric Conditions
$U_a = M_a A_a$	Calculate Cruising Speed at Atmospheric Conditions
$T_{ta} = T_a \left(1 + (\gamma - 1)\frac{M_a^2}{2}\right)$	Calculate Total Temperature at Atmospheric Conditions
$P_{ta} = P_a \left(1 + (\gamma - 1)\frac{M_a^2}{2}\right)^{\gamma/(\gamma-1)}$	Calculate Total Pressure at Atmospheric Conditions

Diffuser

$A_{in} = \frac{\dot{m}(1+\alpha)}{\rho_a U_a g}$	Calculate Diffuser Area Inlet
$P_{t2} = P_{ta}$	Diffuser Outlet Total Pressure
$T_{t2} = T_{ta}$	Diffuser Outlet Total Temperature

Fan

$P_{t7} = P_{t2} * \pi_f$	Fan Outlet Total Pressure
$T_{t7} = T_{t2} * (\pi_f^{(\gamma-1)/\gamma})$	Fan Outlet Total Temperature

Fan Nozzle

$P_9 = P_a$	Fan Nozzle Outlet Pressure
$P_{t9} = P_{t7}$	Fan Nozzle Outlet Total Pressure
$T_{t9} = T_{t7}$	Fan Nozzle Outlet Total Temperature
$M_9 = \sqrt{\frac{2}{\gamma-1}\left\{\left(\frac{P_{t9}}{P_9}\right)^{(\gamma-1)/\gamma} - 1\right\}}$	Fan Exit Mach Number
$T_9 = T_{t9}/(1 + \frac{\gamma-1}{2} * M_9^2)$	Fan Exit Temperature
$A_9 = \sqrt{\gamma R g T_9}$	
$U_9 = M_9 A_9$	

Compressor

$P_{t3} = P_{t2} * \pi_c$	Compressor Outlet Total Pressure
$T_{t3} = T_{t2} * (\pi_c^{(\gamma-1)/\gamma})$	Compressor Outlet Total Temperature

Burner

$P_{t4} = P_{t3}$	Burner Outlet Total Pressure
$\dot{m}_f = \frac{\dot{m} C_p (T_{t4} - T_{t3})}{\Delta H}$	Fuel Mass Flow Rate

Turbine

$T_{t5} = T_{t4} - T_{t3} + T_{t2} - \alpha * T_{t7} + \alpha * T_{t2}$	Turbine Outlet Total Temperature
$P_{t5} = P_{t4} * (\frac{T_{t5}}{T_{t4}})^{\gamma/(\gamma-1)}$	Turbine Outlet Total Pressure

Primary Nozzle

$P_{t8} = P_{t5}$	Primary Nozzle Outlet Total Pressure
$T_{t8} = T_{t5}$	Primary Nozzle Outlet Total Temperature
$M_8 = \sqrt{\frac{2}{\gamma-1}\left\{\left(\frac{P_{t8}}{P_8}\right)^{(\gamma-1)/\gamma} - 1\right\}}$	Fan Exit Mach Number
$T_8 = T_{t8}/(1 + \frac{\gamma-1}{2} * M_8^2)$	Primary Nozzle Exit Temperature
$A_8 = \sqrt{\gamma R g T_8}$	
$U_8 = M_8 A_8$	

Thrust and TSFC Calculation

$Thrust = \frac{\dot{m}*(U_8-U_a)+\alpha*\dot{m}*(U_9-U_a)}{g}$	Calculate Thrust
$TSFC = \frac{\dot{m}_f h_c}{Thrust}$	Fuel Efficiency

The TSFC, thrust specific fuel consumption, is an engineering term to describe the fuel efficiency of a turbine engine. It is a value used to often compare the efficiency of different sized jet engines. Contrary to the equivalent for a car, miles/gallon of fuel, TSFC improves as it decreases in value. In order for this jet engine to be economically viable, the TSFC must be less than 0.5 $\frac{lbm}{hr-lbf}$. The burner fuel heating value (ΔH) is 17800 (Btu/lbm) and the burner exit total temperature is 2500 ($^\circ R$).

Report

Turn in a report with the following sections:

1. Title Page with Summary. The Summary should be short (less than 50 words), and include a table of results.
2. Procedure: A table with the analysis variables, design variables, analysis functions and design functions.
3. Results: Briefly describe the results of optimization (values). Include a table of all calculated values.
4. Discussion of Results: Briefly discuss the optimum and design space around the optimum. Do you feel this is a global optimum?
5. Graphs: A 'zoomed out' contour plot showing the design space (both feasible and infeasible), A 'zoomed in' contour plot of the design space (mostly feasible space) of π_c vs π_f. Show another graph illustrating the optimum region of α vs $\dot{m}$.
6. Further Analysis: By looking at the available graph for π_c vs π_f, discuss what would be the optimum thrust generated if TSFC were not an issue (Oil has dropped in price). Is this the global optimum for no limitation on TSFC? Try different TSFC constraints, does the TSFC constraint change this maximum possible thrust in a feasible design space? Include a table or plot to justify your conclusion.