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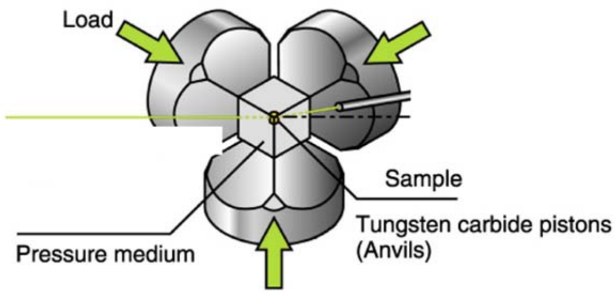
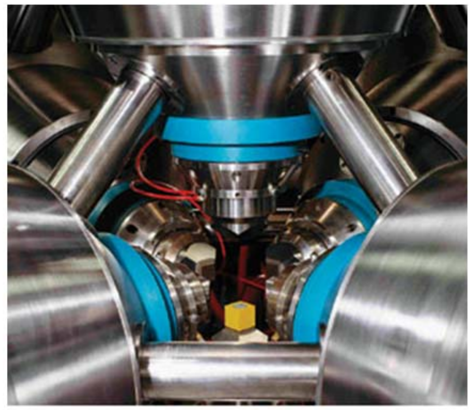
ME 575: Accelerated Life Model

In the manufacturing of synthetic diamond for oil drilling cutters, diamond grit is compressed inside a pressure-transmitting medium at pressures near 1,000,000 psi in cubic presses. The cubic press contacts the pressure medium with tungsten carbide anvils. During the life of an anvil, it will be cycled hundreds of times at different pressures to sinter diamond.



http://jolisfukyu.tokai-sc.jaea.go.jp/fukyu/mirai-en/2010/img/honbun/large/4-2.jpg



http://www.worldoil.com/uploadedimages/Issues/Articles/Nov-2004/04-11_diamond-Jensen_fig1.jpg

Optimization Problem

It is desired estimate the distribution of lifetimes for the anvils. By fitting a statistical model to the data, the probability of failure can be predicted for an anvil at a certain number of cycles and a given pressure. To determine the distribution, first the cycles of the anvils at different pressures has to be brought to a single reference pressure. The lifetimes for each anvil at this reference pressure is then fit to the distribution model so as to maximize the likelihood function for the distribution.

Modeling

Finding the life of each anvil as if it had been run at 1500 tons for every run is accomplished using the acceleration factor:

$$AF = \left(\frac{1500}{tonnage}\right)^P$$

Where *AF* is the acceleration factor, *tonnage* is the press tonnage that the anvils were cycled at for a given run, and *P* is the acceleration parameter. The final accelerated life for a given anvil can thus be represented by:

$$AL = \sum_{i=1}^{n} m_i AF_i$$

Where *AL* is the accelerated life, *n* is the number of unique pressures that the anvils were cycled at, m_i is the number of cycles that occurred at a given pressure, and AF_i is the acceleration factor at that pressure.

The accelerated life totals for each anvil are believed to follow a Weibull distribution. The likelihood function for the Weibull distribution is given by:

$$LIK = \left[\prod_{i=1}^{r} f(t_i)\right]\left[\prod_{j=1}^{s} R(t_j)\right]$$

Where r is the number of failed anvils, s is the number of surviving anvils, and $f(t_i)$ and $R(t_j)$ are the probability density function (pdf) and reliability function, respectively. These functions are given by:

$$f(t) = \frac{\beta}{t}\left(\frac{t}{\alpha}\right)^{\beta} e^{-(t/\alpha)^{\beta}}$$

$$R(t) = e^{-(t/\alpha)^{\beta}}$$

In the pdf and the reliability function, the time t is equivalent to AL for the failed and surviving anvils, respectively.

Objective

Maximize the likelihood function by varying the acceleration parameter, P, and the Weibull parameters α and β. Remember that the objective itself doesn't represent anything meaningful, only the parameters do. Hint: To maximize the likelihood function, minimize the negative natural log of the likelihood function (this allows you to use a summation in place of the product in the likelihood function).

Data

Because this problem was originally developed using proprietary data, simulated data for 20 anvils was generated, as given below. Due to difficulties in creating simulated data, the resulting optimal acceleration parameter, P, will be found to be positive, even though for it to have real world meaning it would need to be negative. In the simulated data, those with a condition code of "0" did not fail during the test (use the reliability function instead of the pdf for these). There were 3 different pressures that the anvils were cycled at during their life: 1500 tons, 1900 tons, and 1800 tons. (Data continued on next page.)

Anvil ID	1500	1900	1800	Condition
1	2247	1497	243	0
2	1871	1302	722	0
3	193	1977	1134	0
4	138	53	36	1
5	246	98	61	1
6	626	320	77	1
7	239	64	94	1
8	322	105	106	1
9	660	259	170	1
10	647	223	200	1
11	487	78	250	1

12	1099	414	301	1
13	515	39	313	1
14	1177	448	318	1
15	670	92	362	1
16	1164	286	489	1
17	1633	528	545	1
18	1427	262	698	1
19	1514	21	1028	1
20	2457	378	1284	1