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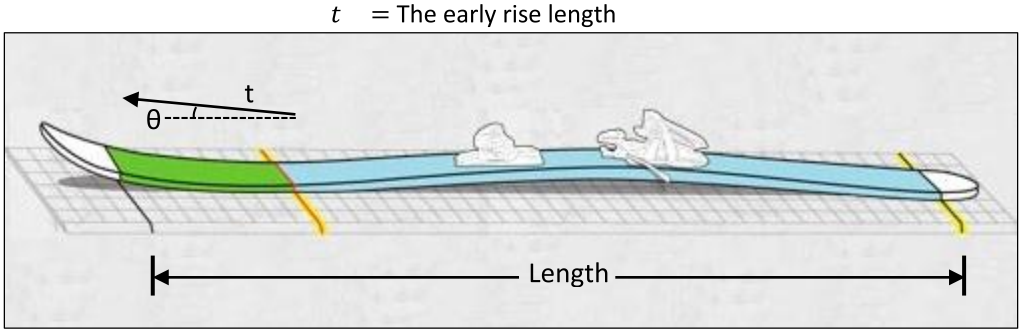
ME 575
Optimization Project #1

Optimization of a Powder Ski

The optimal design of a powder ski is determined by optimizing the width of the ski as well as the profile of the ski when viewed from the side. A powder ski should be wide enough to stay near the surface of deep snow, and have an early risen tip that enables the ski to plane upward in the snow. The location of this early rise is somewhere between the widest point at the tip of the ski and the foot. This report examines the length of the ski as the distance from the two widest points of the ski (tip and tail) and looks at the rise angle as the angle formed by the ground, and the line that extends from the point of early rise to the point of maximum tip width. The extreme tip and tail ends of the ski are ignored in this report.

The variables:

$$\begin{aligned} tip &= \text{The width of the widest part of the front of the ski} \\ waist &= \text{The width of the narrowest part of the ski} \\ tail &= \text{The width of the widest part of the back of the ski} \\ \theta &= \text{The early rise angle} \\ t &= \text{The early rise length} \end{aligned}$$



The profile of a ski when viewed from the top can be looked at as an arc of a circle. This arc has a certain radius that influences the ability of the ski to be turned. The correlating performance aspects of the ski can be calculated with equations of a circle.

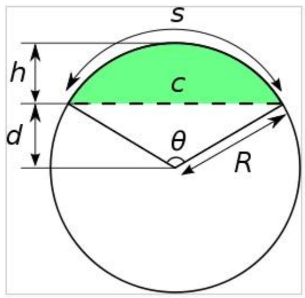


Figure 1: Geometry of Circle Segments

$$h = \frac{1}{2}\left(\frac{tip}{2} - \frac{tail}{2}\right) + \frac{tail}{2} - \frac{waist}{2}$$

$$R = \frac{h}{2} + \frac{l^2}{8h}$$

$$d = R - h$$

$$\phi = 2\text{acos}\left({}^{d}/_{R}\right)$$

The surface area of the entire ski is calculated by subtracting the areas of the segments on each side of the ski from the area of the ski calculated by looking at the ski as a trapezoid.

$$A_{arc} = (\phi - sin(\phi))\frac{R^2}{2}$$

$$A_{tri} = l\frac{(tip + tail)}{2}$$

$$A_{ski} = A_{trapezoid} - (2\,A_{arc})$$

The early rise of the ski decreases the surface area of the ski. The following calculations find how much of the area will be gone because it is part of the tip. The intermediate values of y1 and y2 look at the small angles in the arc as triangles to simplify the equations. The area of these triangles are subtracted from the horizontal length of the tip rise multiplied by the tip width. This value is then subtracted from the ski area to obtain the area of the ski that is on the ground (surface area).

$$l_{rise} = t * \cos(\theta)$$

$$y_1 = \sqrt{R^2 - \frac{l}{2}^2}$$

$$y_2 = \sqrt{R^2 - \left(\frac{l}{2} - l_{rise}\right)^2}$$

$$\Delta y = y_2 - y_1$$

$$A_{tip} = tip * l_{rise} - \Delta y * l_{rise}$$

$$A_{surf} = A_{ski} - A_{tip}$$

The intermediate float is calculated to include the angle of the early rise as well as the length of the early rise, placing greater importance on the length.

$$float = \theta^{10} * t * \cos(\theta)$$

Desirability equations are created with bounds for the maximum and minimum values of width, along with a value chosen by an expert to be the most desirable for width. The programmer must create 2nd order polynomial equation with length as the input, and a value between 0 and 1 as the output. 1 is the most desirable and 0 is the least. The apex should be 1 and the high and lows of the range should be 0. The desirability equations will be multiplied together to make a desirability factor.

Desirability of Waist

Lower Bound(m)	Apex(m)	Upper Bound(m)
0.1	0.1075	0.115

Desirability of Tip

Lower Bound(m)	Apex(m)	Upper Bound(m)
0.12	0.14	0.16

The results of both of these desirability equations should be multiplied together. This product should also be multiplied with A_{surf} and $float$, resulting in the greatness factor to be maximized.

The tip should be greater than the waist. l_rise should be less than a 40% of the total length. Radius must be less than 25 meters and the $^{tip}/_{tail}$ ratio should be 1.05.