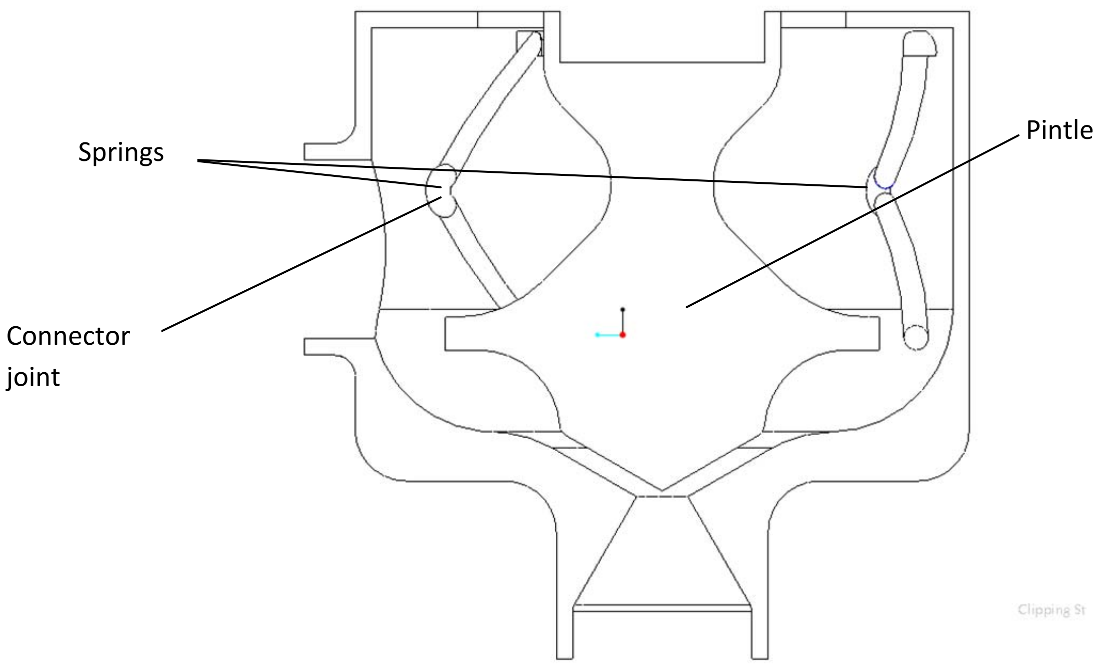
Helical Spring Problem

In the design of a control valve for a liquid fueled rocket, a helical spring is used to secure the pintle and allow it to actuate in and out of the valve orifice (see figure 1).



We wish to optimize the helical spring to achieve the smallest design while minimizing the amount of axial force necessary to actuate the valve. The stress in the spring legs should be below the yield stress and the design must conform to manufacturing constraints.

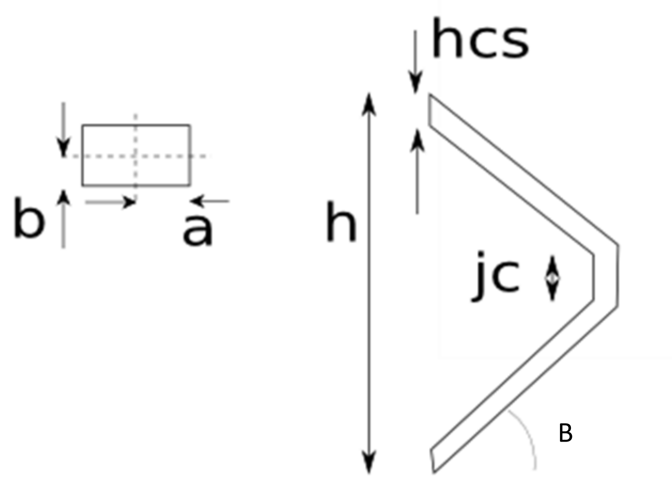
The variables that define the design of this helical spring are:

- *a* – half the height of spring leg cross section in inches
- *b* – half the width of spring leg cross section in inches
- *β* – initial angle of the spring in radians
- Height – straightened length of spring legs in inches

The material used for this spring will be electron beam melted (EBM) titanium. This material has the following properties as defined by the manufacturer:

- Modulus of Elasticity, E – 17.4e6 psi
- Poisson's Ratio, ν – 0.32
- Yield Strength , *Sy* – 68,000 psi
- Ultimate Strength, *Sut* - 135,000 psi

The resolution of the EBM manufacturing process is 0.04 inches, so no dimension can be smaller than that. The recommended gap between walls for the EBM is 0.08 inches. The outer diameter of the valve is 1.5 inches and the wall thickness is set at 0.04 inches. There is a connector where the spring legs fold back on themselves. This connector length, *jc,* is set at 0.08 inches.



For the objectives we will define size as the sum of the height and the outer diameter. We want to set an upper limit on the force of 3 lbs to cause a deflection δ of 0.1 inches. The spring must also not be allowed to clash (reach its solid height).

The force of the spring *F* can be calculated by:

$$F = k_{tot}\delta$$

Where *Ktot* can be calculated by combining the spring constant, *ks*, for each leg. The individual spring constant is multiplied by the number of legs (3) and divided by 2 to account for the fold back nature of the spring:

$$k_{tot} = \frac{3}{2}k_s$$

ks is defined as:

$$k_s = \frac{4k_k G}{D_{coil}^2 L}$$

kk is a special parameter used to calculate the stress in a spring;

$$k_k = ab^3\left(\frac{16}{3} - 3.36\frac{b}{a}\left(1 - \frac{b^4}{12a^4}\right)\right)$$

G is the shear modulus of elasticity:

$$G = \frac{E}{2(1+\nu)}$$

L is the length of a single leg of the spring and can be calculated by:

$$L = \frac{\delta_z - h_{cs}}{\sin \beta}$$

δz is the change in height over the length of one leg of the spring:

$$\delta_z = \frac{h - jc}{2}$$

The diameter of the coil is calculated using a mean radius value, *rlm*, which is defined as:

$$r_{lm} = \frac{r_{li} + r_{lo}}{2}$$

The shear stress in the spring, τ, may be calculated using the following formula:

$$\tau = \frac{k_{tot} * \delta}{6} \left(\frac{D_{coil}}{2Q} + \frac{1}{4ab} \right)$$

Where Q is a parameter defined as:

$$Q = \frac{8a^2b^2}{3a + 1.8b}$$