Optimizing an Original Flywheel Design

Wyatt Felt, Clifton Dudley, Warren Robison

Leonardo da Vinci once proposed a flywheel design with a variable moment of inertia by attaching weights to chains. The initial moment of inertia was low, making it easy to start the flywheel. As it sped up, however, the centripetal force pushed the weights out and made the flywheel resist change in speed and store much more energy. We include a picture of the design here.

We propose a slight variation of da Vinci's design, replacing the chains with springs. This increases the energy that can be stored in a flywheel as some of the energy goes into stretching the springs. We will try to maximize the additional energy per unit mass that can be obtained from this design. Some simplifying assumptions that we make are:

- The weights move only in a radial direction (i.e. we entirely ignore gravity and its effect on the travel path of the weights).
- We assume all of the weights move in and out at the same rate.
- We assume there is something to prevent the weights from extending past the maximum radius allowed.
- We ignore the energy stored by anything but the weights and the springs.
- To simplify computation, we assume the weights are cube-shaped.

The maximum amount of energy that can be stored in this design is:

$$Energy = \frac{1}{2}n\left(kx^2 + m\omega^2 {r_f}^2\right)$$

where n equals the number of weights (and springs), k is the spring stiffness, m equals the mass of each weight, ω equals the maximum angular velocity, r_f equals the distance of the weights to the axis of rotation, and $x = r_f - r_i$, the distance the springs are stretched. We will compare this design to a normal flywheel with the weights attached at r_i, the initial radius. Thus our objective function is of the following form:

$$Energy\ Ratio = \frac{1}{2}n\left(kx^2 + m\omega^2 {r_f}^2 - m\omega^2 {r_i}^2\right)/(n(m + .01))$$

The .01 in the denominator is essentially a penalty function representing the added weight of the springs. It may appear from our objective function that the energy ratio does not depend on n. However, the size and number of weights directly affect the initial radius, as they would overlap at a smaller radius (a physical impossibility). The smallest distance from the center of rotation that the weights can be at is defined by the inner apothem of an n-sided polygon. We refer you to Wikipedia for the theory of the apothem, but give the equation here:

$$a_i = \frac{1}{2}s * \tan\left(\frac{\pi(n-2)}{2n}\right)$$

In this equation, s refers to the side length of the polygon, which is the side length of the weights. Likewise, we should mention here that since the equations for energy are computed from the center of mass of the weights, the initial radius is equal to the apothem plus ½ of the length of the weights. Likewise, the final radius is equal to the outer apothem minus ½ of the length of the weights. The outer apothem equals

$$a_o = r * \cos\left(\frac{\pi}{n}\right)\ ,\ r = \text{maximum radius}$$

Parameters

Density: We will assume the weights are made of lead (11430 kg/m^3)
Maximum Radius: The maximum radius cannot exceed .5 meters.
Maximum Angular Velocity: The flywheel spins at 3600 rpm.

Variables

Dimensions of Weight: the weights are cubes, and the side length must exceed 5mm.
Number of Weights and Springs: there must be at least 1.
Spring Constant: Must be positive and cannot exceed 500 N/m.

Constraints

Forces: The centripetal force must exceed the spring force over the entire path of motion.
(Hint: since both forces are linear in nature and the centripetal force exceeds the spring force at the initial radius, it is only necessary to enforce this constraint at the final radius).
Path of Motion: The weights must travel a positive distance.
Physical Constraints: The apothems and length of the weight must be positive yet less than the maximum radius. No weights can overlap.