ME 575: Fin Array Heat Transfer

For most high performance Central Processing Units (CPUs) and graphics processors it is necessary to use a heat sink on top of the chip to dissipate the heat generated by the chip. The heat sink does this by transferring heat from the chip to the air. A common design is to machine an array of fins out of a solid piece of aluminum to increase the heat transfer rate of the sink. In this problem we will generate an optimal design for an array of square fins on a heat sink.

Optimization Problem

In this problem, we will consider a square heat sink (W_s , W_s) with square fins (w_f x w_f). Our goal is to develop a heat sink that will be able to transfer all the heat generated by the chip into the air. The ambient air temperature is $T_{inf} = 25°C$, and the forced convection heat transfer coefficient of the air is assumed to be constant at $h = 100\frac{W}{m^2K}$. For simplicity, we will not analyze the heat transfer from the chip to the base of the fins. Instead we will assume that when the chip is at its maximum operating temperature, the temperature of the base of the fins will be $T_{base} = 88°C$. Our heat sink must be able to dissipate at least $E_{chip} = 200W$ when the base of the fins is at the temperature given above.

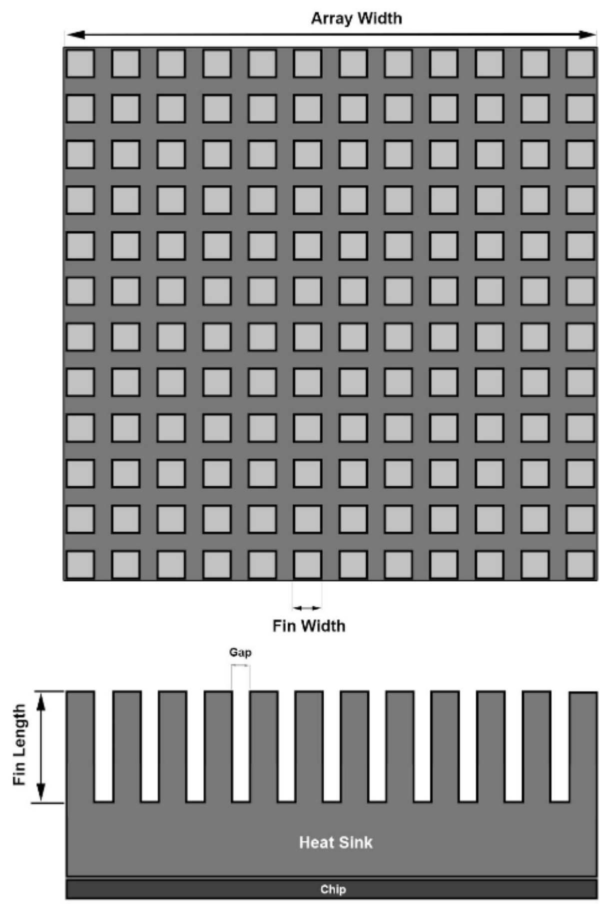

Exhibit good practice in developing a correct model of the system, optimizing the model and correctly interpret the results.

Modeling

An understanding of the principles of heat transfer is necessary to successfully model this system. It is easiest to begin with the energy balance equation:

$$E_{in} - E_{out} + E_{gen} = E_{st}$$

where all quantities are time rates of energy transfer or storage. We are interested in the steady state operation of the heat sink, so the storage and generation terms are zero. Thus, the heat equation reduces to the following:

$$E_{in} - E_{out} = 0$$

We assume that all of the heat generated in the chip is transferred out through the heat sink. This means

$$E_{in} = q_{in} = 200\ W$$

E_{out} is the rate at which heat leaves the heat sink by convection from the fins and from the portion of the base area that is not covered by fins (i.e. the area at the bottom of the gaps).

$$E_{out} = q_{fin} n_{fins} + q_{gaps}$$

The heat transfer from an individual square fin is:

$$q_{fin} = \frac{M\left(\sinh(mL_{fin}) + \left(\frac{h}{mk}\right)\cosh(mL_{fin})\right)}{\left(\cosh(mL_{fin}) + \left(\frac{h}{mk}\right)\sinh(mL_{fin})\right)} \qquad M = \theta_b \sqrt{hP_{fin} k A_{fin}} \qquad m = \sqrt{\frac{hP_{fin}}{kA_{fin}}}$$

$$\theta_b = T_{base} - T_{inf} \qquad P_{fin} = \text{Cross-sectional perimeter of fin} \qquad A_{fin} = \text{Cross-sectional area of fin}$$

Where k is the conductive heat transfer coefficient of aluminum, $k = 237\ \frac{W}{m*K}$. Convection due to the gaps is then defined as follows:

$$q_{gaps} = hA_{gaps}(T_{base} - T_{inf})$$

Objective

The objective of this problem is to minimize total cost while keeping the chip at or below its operating temperature. This means designing an array that will dissipate all of the heat generated by the chip when the chip is at its maximum operating temperature. If a particular heat sink design is able to dissipate more heat than this at the maximum operating temperature, then the actual operating temperature will be kept lower than the maximum and the design is still acceptable.

In this case, we will consider material and manufacturing costs of the fin array (we neglect the material cost of the base). For Aluminum, the cost is $1.25/kg, and the density is 2800 $\frac{kg}{m^3}$. The cost for each cut needed to make the heat sink is $0.25. Due to manufacturing restrictions in our factory, the gap between fins must be greater than 1.5 millimeters, and the fins themselves must also be at least 1.5 millimeters wide. For the heat sink to fit in the computer, the width of the heat sink must be no larger than 5 cm. For the same reason, the length of the fins (L_{fin}) must be less than 3 cm.

Report

Provide an executive summary of your results including a table of the values of the design variables and objective function. Include a discussion on how you arrived at the optimal solution. Also, state whether you think your solution is a global solution. Your report should also include the following:

1. A contour plot of the solution with the optimal value marked and the feasible region shaded.

2. An appendix including your model file.