University of Utah
50 Central Campus Dr,
Salt Lake City, UT 84112

April 23, 2018

Dr. John Hedengren
350 CB Brigham Young University
Provo, UT 84606

Dr. Hedengren,

Introducing variable energy prices into the power grid would reduce the consumer side energy consumption. Smart automation of buildings with variable energy price signals would also allow for cost savings for the residents, which paves the way for deeper studies related to the power grid.

In this work, a semi-empirical model for the heat transfer through a house in Salt Lake City was implemented. Moreover, a dynamic real-time optimization problem for minimizing the energy cost of the house was implemented and solved. Results show significant cost reductions in the optimized case with variable energy price (Time of Use pricing structure) versus the non-optimized case.

Respectfully,

Derek Machalek
Kevin Ellingwood
Moataz Sheha
Jake Tuttle

Project Highlights

- Semi-empirical, first-order model is developed to simulate indoor temperature of residential house.
- Dynamic optimization is used to minimize cost of heating and cooling requirements by over 15%
- Indoor temperature is most sensitive to changes in ambient atmospheric temperature
- Optimization of house energy usage is valid if weather forecast is within 3 K.

# Dynamic optimization of home heating and cooling costs

Dynamic Optimization

Project Report

Derek Machalek

Kevin Ellingwood

Moataz Sheha

Jacob Tuttle

## *Abstract*

Dynamic optimization is used to minimize the cost of heating and cooling of a residential house. A semi-empirical model was developed to model the indoor home temperature as a function of ambient atmospheric temperature, direct normal irradiance (DNI), and heating/cooling requirement. The energy cost was designated as the objective function to be minimize and heating and cooling were both assumed to be electrical. The cost is optimized using the forecast temperature over a 24-hour period. A sensitivity analysis of indoor temperature was carried out and established ambient temperature as the most influential factor in modeling. Dynamic optimization of the house energy system resulted in an approximate 15.5% savings in heating and cooling over an optimal control system with a fixed set point within the pre-defined bounds of comfort. Additional analysis was carried out to determine the effect that inaccurate weather forecasting has on the optimization. If the random error between the ambient temperature and forecasted temperature is within 3 K, the optimization is useful for cost savings. For forecasts where the temperature is inaccurate over 3 K, the optimization is not valid, and costs increase. Regarding systematic error, a high biased error in the forecast quickly negates the benefits of dynamic optimization, while a low biased error in the forecast diminishes, but does not negate, the benefits of dynamic optimization. These findings underscore the importance of an accurate forecast for dynamic optimization of home heating.

## *Introduction/Literature Review*

The energy required to heat and cool houses makes up a considerable portion of the total energy consumption in houses, and as a result, the monthly energy bill. In the summer months, the energy inside of a home is continually impacted by the energy gained due to solar radiance, convection and conduction from high ambient temperatures through the walls, windows, doors, roof, and floor, as well as energy removed through the controlled air conditioning system. In most houses, the control scheme of inside temperature is usually simplistic with an upper and lower set point which establishes a temperature range. The heating or cooling is undertaken to maintain the temperature within the range of the set point limits. On top of the continuous temperature change and variable pricing, the residents inside the home have temperature set point constraints based on the time of day (i.e. when they are at home or at work). Advanced control techniques offer possible routes forward to minimize the energy requirements by modifying the controlled temperature over times of the day. In this project, we utilize dynamic optimization techniques to minimize the energy cost of heating or cooling of residential housing. The objective function must meet the temperature constraints based on the preferences of the residents. The cost of energy is modeled based on a time-of-use pricing schedule, which incentivizes off-peak energy usage.

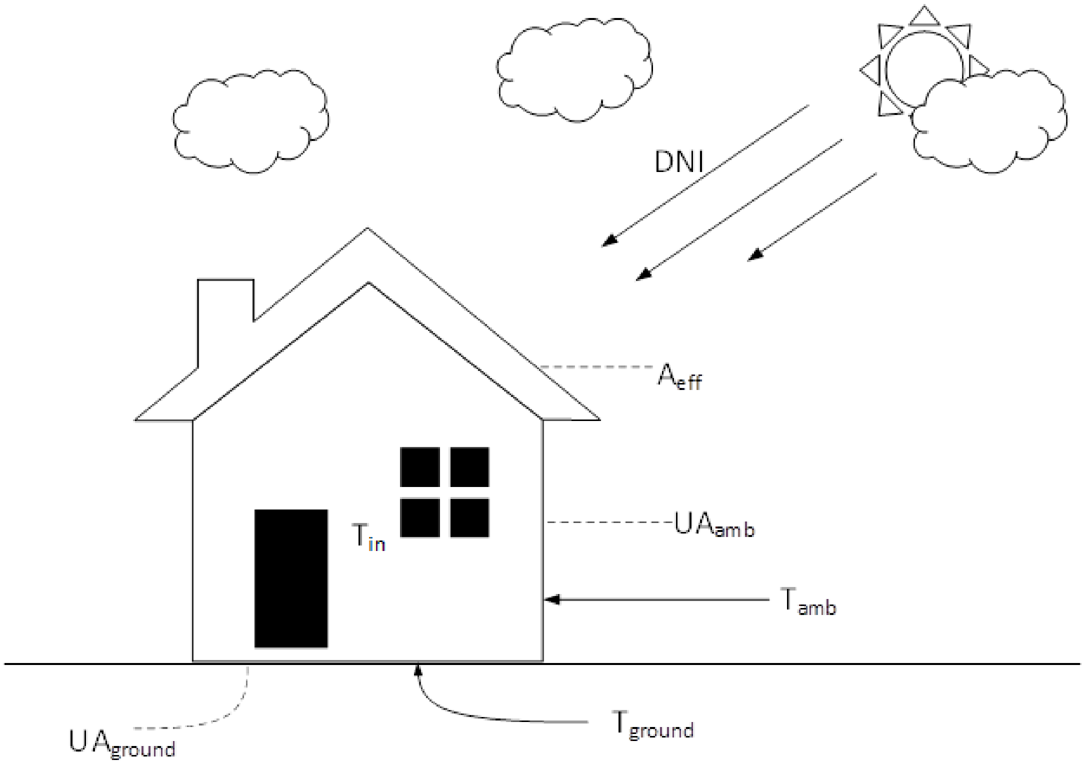


*Figure 1: Schematic of heat transfer mechanisms and variables effecting cooling energy requirements in a house.*

A diagram of the system is shown in Figure 1. The indoor cooling requirements of the house are due to several heat sources, sinks, and heat transfer mechanisms. The house gains heat from solar irradiance modeled as a heat flux using direct normal irradiance (DNI) data readily available. Energy is transferred between the house and surrounding ambient atmosphere via convection, conduction, and radiation. The internal cooling and heating systems act as a heat sink and source, respectively. The heating and cooling requirements are thus responsible for electrical costs.

*Literature Review*

Work performed by Cole and Powell[1] describes the tremendous opportunity associated with optimization of residential cooling. They explain how in the ERCOT (Electric Reliability Council of Texas) grid, over 50% of the total electrical load during the summer of 2011 was due to residential homes, with the homes' primary load being air conditioning systems. Using a reduced order model of a residential home, their work was able to realize a reduction in peak energy consumption from the air conditioning unit by an average of 70%, reducing operating costs by 60% using model predictive control techniques. A major

[1] W. J. Cole, K. M. Powell, E. T. Hale and T. F. Edgar, "Reduced-order residential home modeling for model predictive control," *Energy and Buildings*, no. 74, pp. 69-77, 2014. c

difference in their work compared to what is proposed in this project is the inclusion of solar radiation in the model. This fact contributes to the overall difference in methods employed by Cole and Powell and those proposed for this project, being that the former uses reduced-order models derived from a dataset produced by EnergyPlus for model development, and the latter will use strictly first principles governing equations to model the system. Due to the large amount of energy that can be potentially deposited due to solar radiation, especially through south-facing windows of a home, it is expected that this being included in the model will be very important to accuracy and results.

Additionally, work performed by Sheha et al.[2] presents a simple model for optimizing the cooling energy cost in residential houses through dynamic real-time optimization (D-RTO) under four different electricity pricing structures; flat pricing (FP), time-of-use (TOU), critical peak pricing (CPP), and real-time pricing (RTP). The work demonstrates that dynamic optimization can decrease the energy cost and shift the peak energy consumption periods towards off-peak price hours which paves the way towards incorporating energy storage systems (e.g. battery) and distributed energy resources (e.g. solar panels). Results showed that the four pricing structures were ranked in terms of percentage energy cost reduction as follows: CPP > FP > TOU > RTP. While in terms of percentage energy consumption reduction, they were ranked as follows: FP > TOU > CPP > RTP. On a big community scale of 6000 houses, the reduction of energy consumption was high and reached over a million kWh for the optimized FP and TOU cases. This is an important result for grid-level studies.

## *Theory/Methods*

The model of the system was developed from scratch and is a semi-empirical model. The model is described in the following sections and presented in a way that is compatible with APMonitor.

*Constants*

Table 1 shows the constants that are used within the model.

*Parameters*

The model parameters seen below, in Table 2, are used for the model equation by Equation 2.

*Variables*

The temperature inside of the home, $T_{in}$, is defined as a control variable that is adjusted by the heating and cooling inputs into the home. The other control variable defined in the model is the $Cost$ variable defined by the cost equation, shown in Equation 3.

[2] M. N. Sheha, K. Rashid and K. M. Powell, “Dynamic real-time optimization of air conditioning systems in residential houses under different electricity pricing structures,” *American Control Conference (ACC)*, 2018 (ACCEPTED).

*Table 1: Model constants.*

| **Constant** | **Value** | **Units** | **Description** |
|---|---|---|---|
| $COP$ | 3.5 | $-$ | Coefficient of performance in house AC unit |
| $\eta_h$ | 0.8 | $-$ | Efficiency of house heating unit |
| $Rate$ | 0.1122 | $\frac{\$}{kWh}$ | Cost rate of electricity usage |
| $c_p$ | 1 | $\frac{kJ}{kg*K}$ | Heat capacity of air in house |
| $V$ | 280 | $m^3$ | Volume of air space in house |
| $\rho$ | 1.23 | $\frac{kg}{m^3}$ | Density of air at ambient air temperature |

*Table 2: Model parameters.*

| **Parameter** | **Units** | **Description** |
|---|---|---|
| $B$ | $-$ | Coefficient for ambient temperature loss |
| $C$ | $-$ | Constant coefficient for energy balance |
| $D$ | $-$ | Coefficient for heat flux in form of solar irradiance |
| $E$ | $-$ | Coefficient for empirical term multiplying solar $I_{DN}$ and $T_{amb}$ |
| $I_{DN}$ | $\frac{W}{m^2}$ | Direct normal irradiance |
| $T_{amb}$ | $K$ | Ambient air temperature |
| $Q$ | $W$ | Heating or cooling requirement |

*Model Equations*

The house energy model was initially proposed as a principles first model that utilized ambient convective heat transfer, heat induction from ground temperature, and solar flux as dominant driving forces. The initial model equation was:

$$\frac{dT_{in}}{dt} = B(T_{amb} - T_{in}) + C\left(T_{grond} - T_{in}\right) + D(I_{DN}) - \frac{Q}{\rho V c_p} \tag{1}$$

Data was extracted from BeOpt to get data for a house simulated with no heating or cooling. This data was used to estimate the constants present in the energy model. After preliminary results, the model was not accurate, and estimation did not appear to be fruitful. Also, the ground temperature did not appear to have a substantial effect on the inside temperature so that term was scrapped.

Empirical terms were added to the model and estimation was carried out until a good fit was realized. A good fit was realized with the following model:

$$\frac{dT_{in}}{dt} = B(T_{amb} - T_{in}) + D(I_{DN}) + E(I_{DN})(T_{amb} - T_{in}) + C - \frac{Q}{\rho V c_p} \quad (2)$$

The empirical term $E(I_{DN})(T_{amb} - T_{in})$ was hypothesized to benefit modeling as a secondary effect but has not been used in literature in modeling of house energy usage. Further, heat transfer dynamics between the furnace temperature and the measured home temperature were assumed to be 15 minutes.

From Equation 2, the cost of the house can be estimated from the following:

$$Cost = rate\left(\frac{Q_{cool}}{COP} + \frac{Q_{heat}}{\eta_h}\right) \quad (3)$$

Where,

$$Q_{cool} = Q \; if \; Q > 0$$

$$Q_{heat} = Q \; if \; Q < 0$$

The goal of the project was to minimize energy costs. To do this, a non-flat price structure was needed and the time-of-use (TOU) structure was used for the project. The TOU structure can be seen in Figure 2.

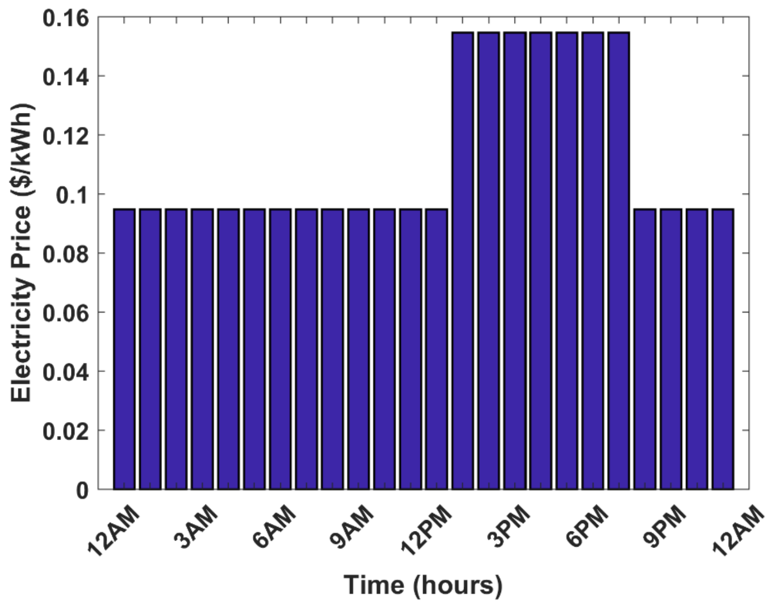


*Figure 2: Time-of-use pricing structure.*

## *Simulation Results and Estimation Results*

Using APMonitor, the parameters $B, C, D$, and $E$ were estimated to fit Equation 2 to simulated BeOpt data for a house with no cooling or heating loads. The results of the parameter estimation are seen in Table 3: Parameter estimation summary. and Figure 3.

Table 3: Parameter estimation summary.

| Parameter | Value | Units | Description |
|---|---|---|---|
| $B$ | 0.25368 | $hr^{-1}$ | Coefficient for ambient temperature loss |
| $C$ | 0.79134 | $\frac{K}{hr}$ | Constant coefficient for energy balance |
| $D$ | 3.582e-4 | $\frac{K \cdot ft^2}{hr \cdot Btu}$ | Coefficient for heat flux in form of solar irradiance |
| $E$ | 9.719e-4 | $\frac{ft^2}{hr \cdot Btu}$ | Coefficient for empirical term multiplying solar $I_{DN}$ and $T_{amb}$ |
| $I_{DN}$ | Variable | $\frac{W}{m^2}$ | Direct normal irradiance |
| $T_{amb}$ | Variable | $K$ | Ambient air temperature |
| $Q$ | Variable | $W$ | Heating or cooling requirement |

The model estimation was based upon physical data generated from BeOpt from NREL. This data was assumed to be accurate as it was generated from software developed by a third party that is nationally renowned for its energy modeling and analysis. The regressed model is shown in Figure 3 along with the measured temperature profile from BeOpt for the month of May. As can be seen the estimated model does a fairly good job of predicting the temperature of the house given the ambient air temperature and solar irradiance for the month of May, which was taken to be used as the standard for modeling.

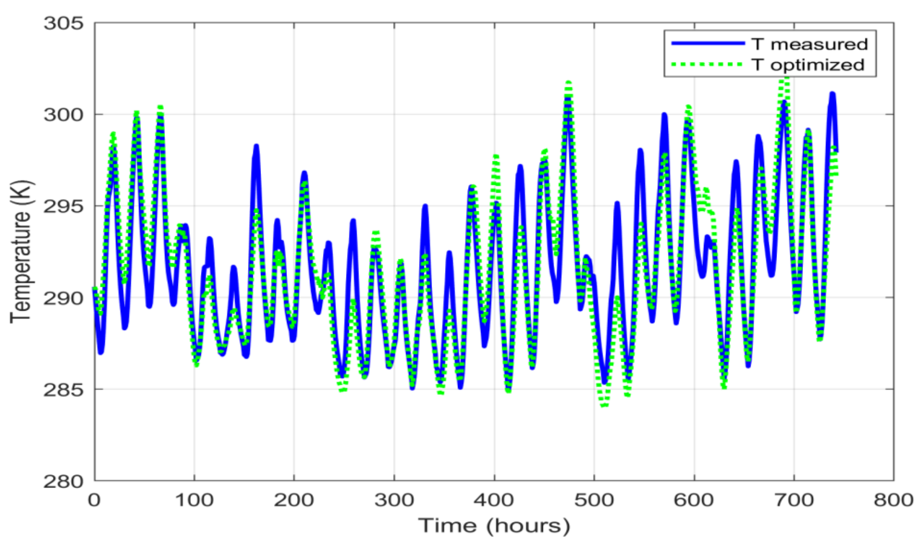


Figure 3: Measured temperatures along with estimated temperatures based upon Equation 1.

In the previous report, control of the system was carried out with only a simple upper and lower bound set point for the house temperature.

## *Sensitivity Analysis and Dynamic Optimization Results*

### *Dynamic Optimization Results*

Once the estimation was complete, the objective cost function (Equation 3) was incorporated into the simulation to minimize the cost by altering the temperature set point of the system. The optimization utilized model predictive control to maintain the temperature within the designated range of set points. The dynamic optimization to minimize the heating and cooling bill of the house is compared to a house with a fixed set point. In this case, the fixed set-point was set to 295 K, the fixed set-point that minimized the heating cost with a set-point. Table 4: Benefits of dynamically optimizing the heating and cooling regime of a house. illustrates the energetic and monetary benefits from optimizing some houses heating and cooling, while staying within the temperature bounds desired by the residents.

*Table 4: Benefits of dynamically optimizing the heating and cooling regime of a house.*

| | **Fixed Set Point** | **Dynamic Optimization** | **Difference (%)** |
|---|---|---|---|
| **Q Total (kW)** | 4031.2 | 3494.1 | -13.3% |
| **Q Cool (kW)** | 1081.3 | 893.5 | -17.4% |
| **Q Heat (kW)** | 2949.9 | 2600.5 | -11.8% |
| **Cost ($)** | 254.5 | 215.01 | -15.5% |

As illustrated in Table 4: Benefits of dynamically optimizing the heating and cooling regime of a house., dynamically optimizing the heating and cooling of the home saves the home 15.5 percent monetarily in addition to reducing the total amount of energy consumed by 13.3 percent. The controller strategy is further illuminated in Figure 4 below which illustrates the heating requirement and internal home temperature for the fixed set point and dynamically optimized homes.

Figure 4 illustrates the dynamically optimized heating of the home to a home with a fixed set point of 295 K. To keep the home at the fixed temperature requires large oscillations in the heating requirement, whereas the dynamically optimized home takes advantage of the wide set point bounds to minimize the heating and cooling peaks. Often undercutting the energy peaks of the fixed set point. The third subplot of Figure 4 illustrates the gap between the cost of home air temperature management for the fixed point and dynamically optimized strategies, with a continuously growing gap between the two. This gap is further clarified in Figure 5, where the relative (percentage) savings from employing dynamic optimization reaches a plateau at around 300 hours, the minimum amount of time the simulation needs to be run to find long term operating benefits of dynamic optimization.

Figure 6 demonstrates the dynamics between the furnace temperature and the home temperature. The slight delay in heating and cooling mandates anticipatory behavior from the central heating system of the home to stay within the requisite temperature bounds.

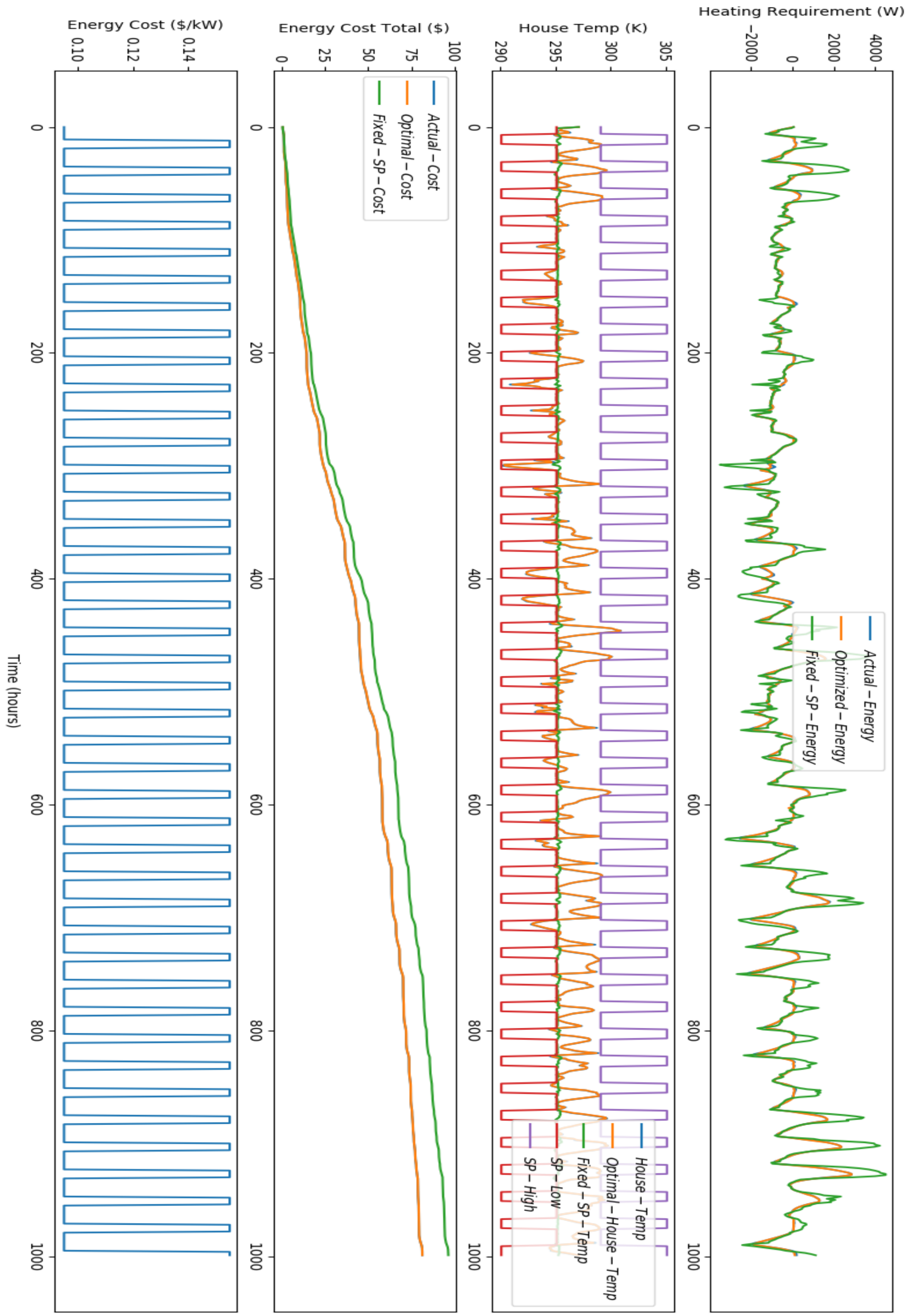


*Figure 4: Temperature Control in a House with Fixed Setpoint vs. with Dynamic Optimization.*

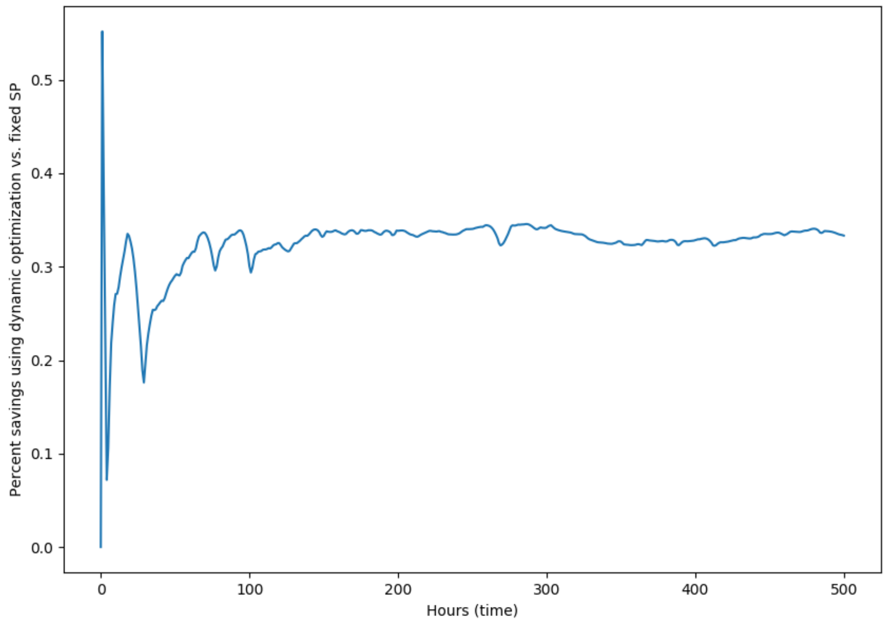


*Figure 5: Percent of Cost Savings over Time for the Month of June.*

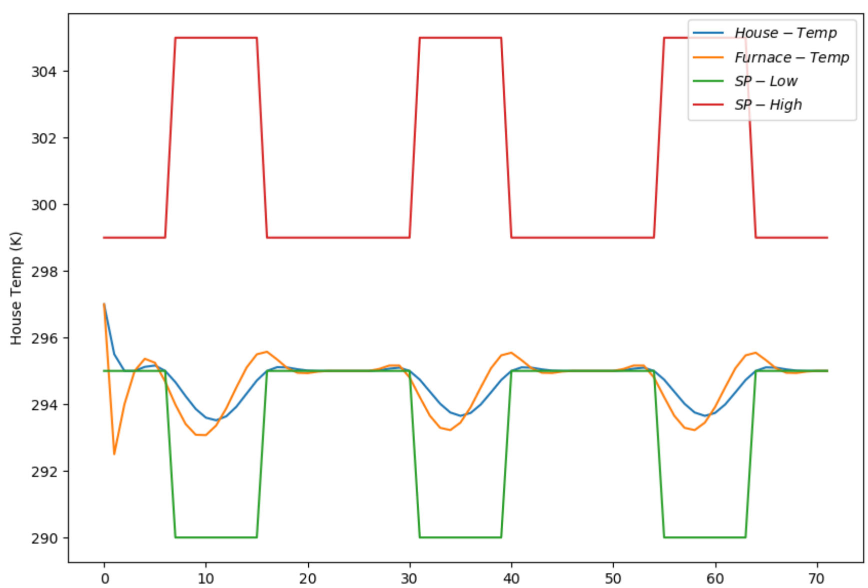


*Figure 6: Dynamics of House and Furnace Temperature for 3 days of simulation.*

*Sensitivity Analysis*

A sensitivity analysis of the house temperature ($T_{In}$) to the model inputs ($T_{Amb}$, DNI, and Q) was performed. The baseline values for the sensitivity analysis are shown in the table below.

*Table 5: Baseline parameters for sensitivity analysis.*

| Parameter | Baseline Value | Units |
|---|---|---|
| **Q** | 1000 | W (Heating) |
| **$T_{Amb}$** | 273 | K |
| **DNI** | 200 | BTU/ft2 |
| **$T_{In}$** | 287.2 | K |

Small changes were made to the input parameters to identify how they impacted the internal temperature of the house.

*Table 6: Sensitivity analysis of house temperature to heating, ambient temperature, and ambient sunlight.*

| Parameter | New Value | Change (%) | $T_{In}$ | Change (%) |
|---|---|---|---|---|
| **Q** | 980 | -2% | 287.15 | -0.01% |
| **Q** | 1020 | 2% | 287.22 | 0.01% |
| **$T_{Amb}$** | 267.54 | -2% | 284.49 | -0.94% |
| **$T_{Amb}$** | 278.46 | 2% | 289.88 | 0.94% |
| **DNI** | 196 | -2% | 287.26 | 0.03% |
| **DNI** | 204 | 2% | 287.11 | -0.02% |

Changes in the house temperature are most sensitive to the changes in the ambient temperature and are minimally sensitive to changes in the input heat and the DNI. Interestingly, the gain for positive and negative changes in the DNI both result in reductions of the internal house temperature, likely due to the empirical term in the model, the product between the DNI and the difference between ambient temperature and house temperature.

*Model vs. Reality Mismatch*

Forecast mismatch analysis was carried out on the ambient temperature data in two different ways; one with random noise around the temperature forecast used for the dynamic optimization with a maximum standard deviation of five, the second way was done with a systematic noise over the temperature forecast (i.e. the noise is the same all the time) with a maximum standard deviation of five in both directions. Figure 7: Model vs. Reality mismatch using random error. and 8 present the results in the form of percentage savings increase or decrease based on the value of the deviation from the weather forecast while performing the model predictive control. The ambient temperature was selected because it was identified as the parameter that the model was most sensitive to. In these models the fixed temperature set point operating condition was raised to 297 Kelvin to increase the window of savings for dynamic optimization.

Figure 7: Model vs. Reality mismatch using random error. shows the impact of random error (randomly generated numbers of a normal distribution with a mean of 0 and the standard deviation specified by the x-axis) on the energy savings with dynamic optimization over a fixed set-point. If the weather forecast has a standard error greater than 3 Kelvin, dynamic optimization is no longer economically favorable compared to operating at a fixed set point. This indicates that the quality of the forecast is essential to the success of this dynamic optimization algorithm, and the accuracy of available forecasts can predict the return on investment of the implementation of a dynamically controlled central air system.

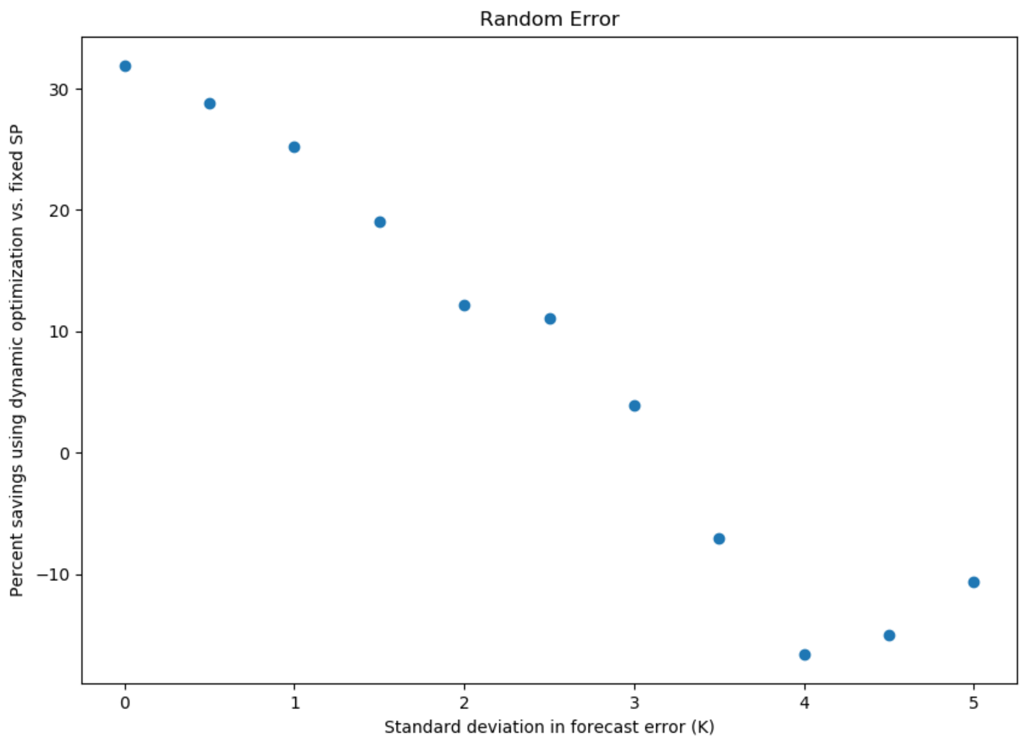


*Figure 7: Model vs. Reality mismatch using random error.*

Figure 8:7 shows the impact of systematic error (a fixed deviation from the actual forecast as dictated by the x-axis) on the savings from dynamic optimization. As expected, no systematic error results in the highest savings. However, the asymmetry of savings with the bias of equal magnitude, but opposite sign, is a surprising result. A negative bias results in small savings asymptotically approaching the savings of the fixed set point, on the other hand, the positive bias quickly degrades to monetary loses with dynamic optimization. When the forecast bias is positive, the dynamic optimization predicts minimal required heating in the future and will operate at a lower set point. Unfortunately, when the house loses more heat than expected, expensive heating must be undertaken (during peak price times) to keep the house within the temperature constraints. At least with a negative bias, the home will lose less heat than expected, but still leverage the use of price mismatch during on-peak and off-peak times. The asymmetric penalty on a biased error indicates that dynamic optimization algorithm could hedge its position by assuming slightly greater than expected energy losses.

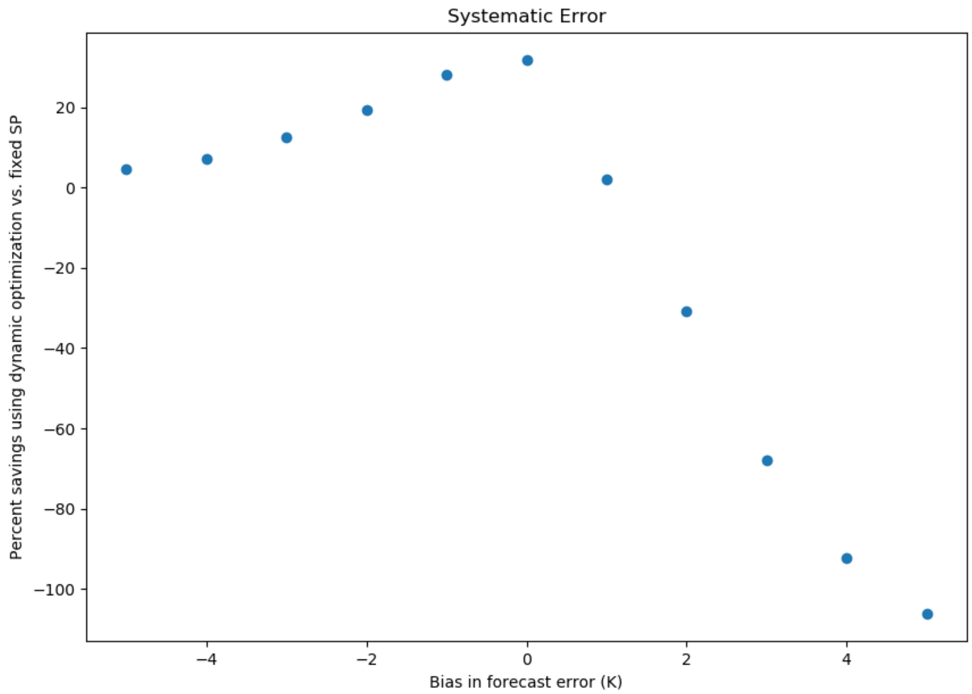


*Figure 8:7Model vs. Reality mismatch using systematic error.*

## *Conclusion*

Dynamic optimization was exhibited to improve the energy cost of a residential house assuming heating and cooling to be both electrical. Optimization resulted in over 15% savings in heating/cooling costs relative to the requirements for a house designated a constant temperature set point. Sensitivity analysis concluded that indoor temperature was a strongest function of the ambient atmospheric temperature and a forecast error study was conducted to demonstrate dynamic optimization's dependence upon an accurate forecast. Optimization of the system is valid if the temperature forecast is within 3 K of the actual temperature; outside of that range, optimization results in an overall increase in energy costs.