Chemical Engineering 436
Special Problem 9

Part 1

Linearize the following function around the point (x,y,z) = (2,1,1):

$$f(x, y, z) = y \cdot (x^2 - yz) + z \cdot e^{x/10}$$

Calculate the error (i.e., $f_{linear} - f_{exact}$) for the following points:

(2,1,1), (2±0.25,1,1), (2,1±0.25,1), (2,1,1±0.25)

Part 2*

A thermocouple bead (T_t) tries to measure the gas temperature T_g, but the gas temperature changes rapidly due to turbulence. Heat is transferred to the thermocouple by convection as expected. Heat transfer to the thermocouple by radiation is also important and can be approximated by the following expression: $q'' = \varepsilon\sigma(T^4_{flame}-T_t^4)$ where ε is the emissivity and σ is the Stefan-Boltzmann constant ($W/(m^2\ K^4)$). T_{flame} is the average soot temperature in the flame, and varies with time as well. The thermocouple bead itself can be approximated as a sphere (e.g. ignore conduction along connecting wires, etc).

a) Write a dynamic model that can be solved for the thermocouple beat temperature (T_t) as a function of time.

b) Linearize the model about a set of steady-state temperatures $\overline{T}_t, \overline{T}_{flame}, \overline{T}_g$, and transform the equation using deviation variables.

*Note: Results from part 2 of this problem will be used in Special Problem 10.