Chemical Engineering 436
Special Problem 10

This is the same problem that you were given for Special Problem 9 (part 2), where you completed parts a-b. You are now requested to complete the rest of the problem. If you missed things on Special Problem 9, look at the answer key on Blackboard.

Some advanced combustion systems operate in a pulsed mode where an acoustic wave resonates in the combustor in order to enhance heat and mass transfer. The frequency of the wave is approximately 100Hz (cycles/sec). As a result, the temperature at any given spatial location in the combustor fluctuates with time. For our purposes, the fluctuations can be approximated as a sine wave with an amplitude of 150^{o}C for the system of interest. We would like to be able to measure the fluctuating temperature at a given point in the reactor with use of a thermocouple. In order to do so, we must develop a dynamic relationship between the temperature of the thermocouple (T_t) and the fluctuating gas temperature T_g. Heat is transferred to the thermocouple by convection as expected. Heat transfer to the thermocouple by radiation is also important and can be approximated by the following expression: $q'' = \varepsilon\sigma(T^4_{flame}-T_t^4)$ where ε is the emissivity and σ is the Stefan-Boltzmann constant ($W/(m^2 K^4)$). The thermocouple bead itself can be approximated as a sphere (e.g. ignore conduction along connecting wires, etc). Also note that no numbers are required until part f).

a) Write a dynamic model that can be solved for the thermocouple temperature as a function of time.

b) Linearize the model about a set of steady-state temperatures $\overline{T}_t, \overline{T}_{flame}, \overline{T}_g$ (Note that we have greatly simplified the actual problem.)

c) Find the transfer functions $T_t'(s)/T_g'(s)$ *and* $T_t'(s)/T_{flame}'(s)$ in standard form.

d) What would be the gain in $T_t'(s)/T_g'(s)$ if the radiation term was negligible? Does this make sense? (briefly justify your response).

e) Assuming that T_{flame} is constant and that the fluctuations in the gas temperature can be approximated as a sine wave as previously mentioned, please provide an analytical expression for the thermocouple temperature as a function of time.

f) Given values for the following parameters (and assuming constant T_{flame}):

$h = 2800$ W/m^{2}K	Steady state thermocouple temperature = 1500K
$\rho_{thermocouple} = 20$ g/cm^3	$T_{flame} = 2000$K
$\sigma = 5.67 \times 10^{-8}$ W/m^{2}K^4	Thermocouple Heat Capacity = 0.4 J/g K
$\varepsilon = 0.8$	Thermocouple diameter = 0.01 cm

Would you recommend use of a thermocouple to measure temperatures in the system described above? Why or why not?