Proportional Integral (PI) Control

The PI Controller

- "Ideal" form of the PI Controller

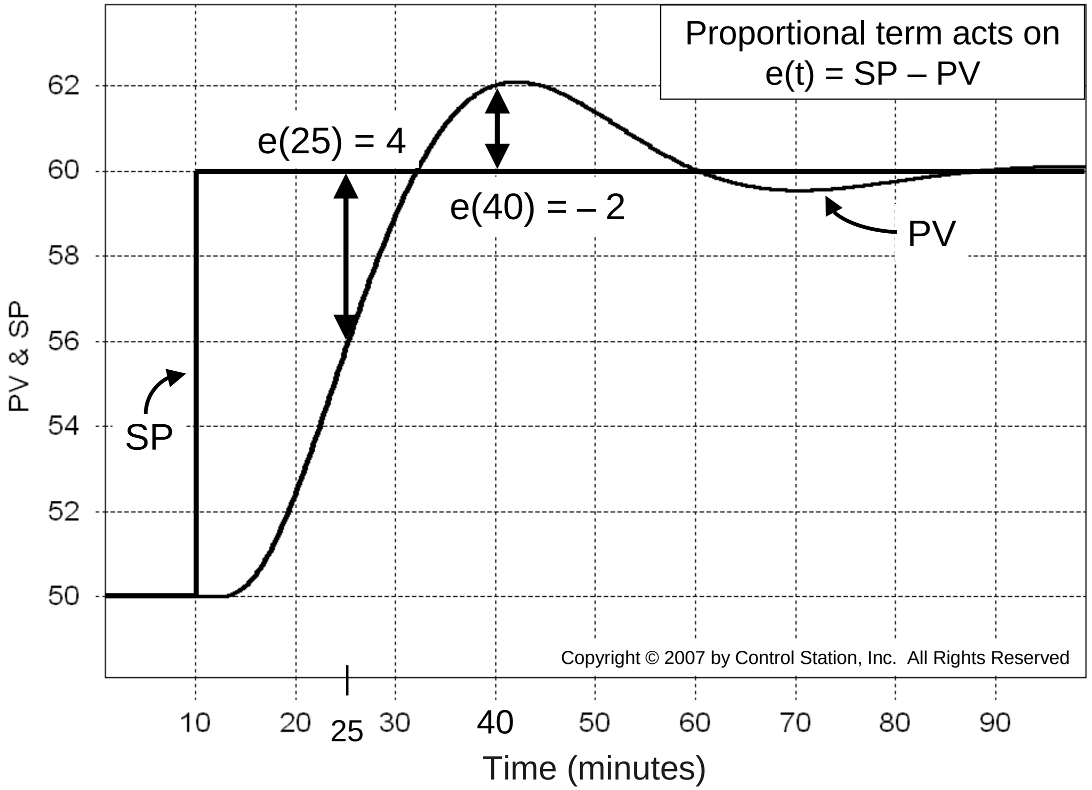
$$CO = CO_{bias} + Kc \cdot e(t) + \frac{Kc}{\tau_I} \int e(t)dt$$

where:

CO	= controller output signal
CO_{bias}	= controller bias or null value
PV	= measured process variable
SP	= set point
$e(t)$	= controller error = SP – PV
Kc	= controller gain (a tuning parameter)
τ_I	= controller **reset time** (a tuning parameter)

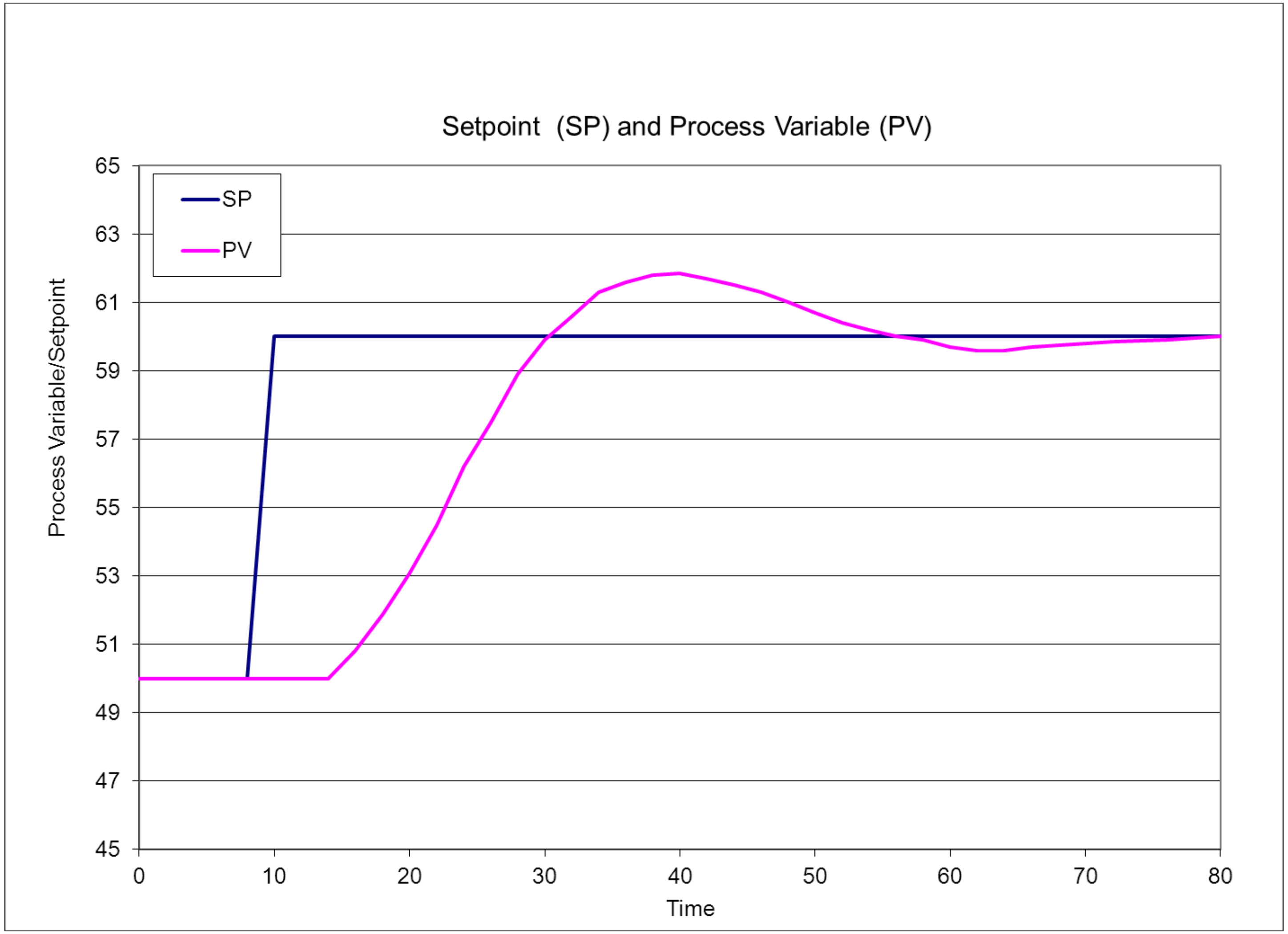
- τ_I is in denominator so smaller values provide a larger weighting to the integral term
- τ_I has units of time, and therefore is always positive

Function of the Proportional Term

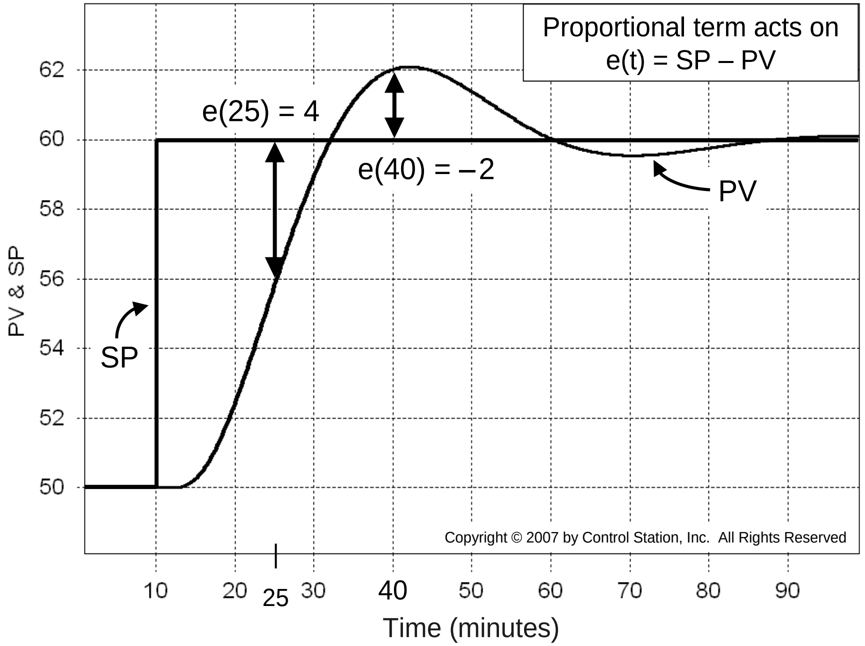


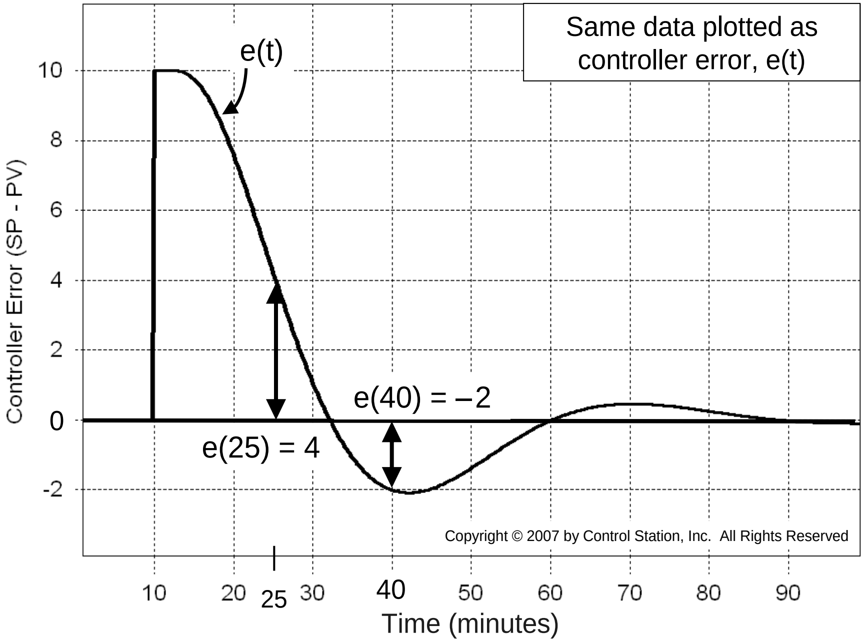
- The proportional term, Kc·e(t), immediately impacts CO based on the size of e(t) at a particular time t
- The past history and current trajectory of the controller error have no influence on the proportional term computation

Class Exercise – Calculate Error and Integral



Control Calculation is Based on Error, e(t)





- Here is identical data plotted two ways
- To the right is a plot of error, where: e(t) = SP – PV
- Error e(t) continually changes size and sign with time

Function of the Integral Term

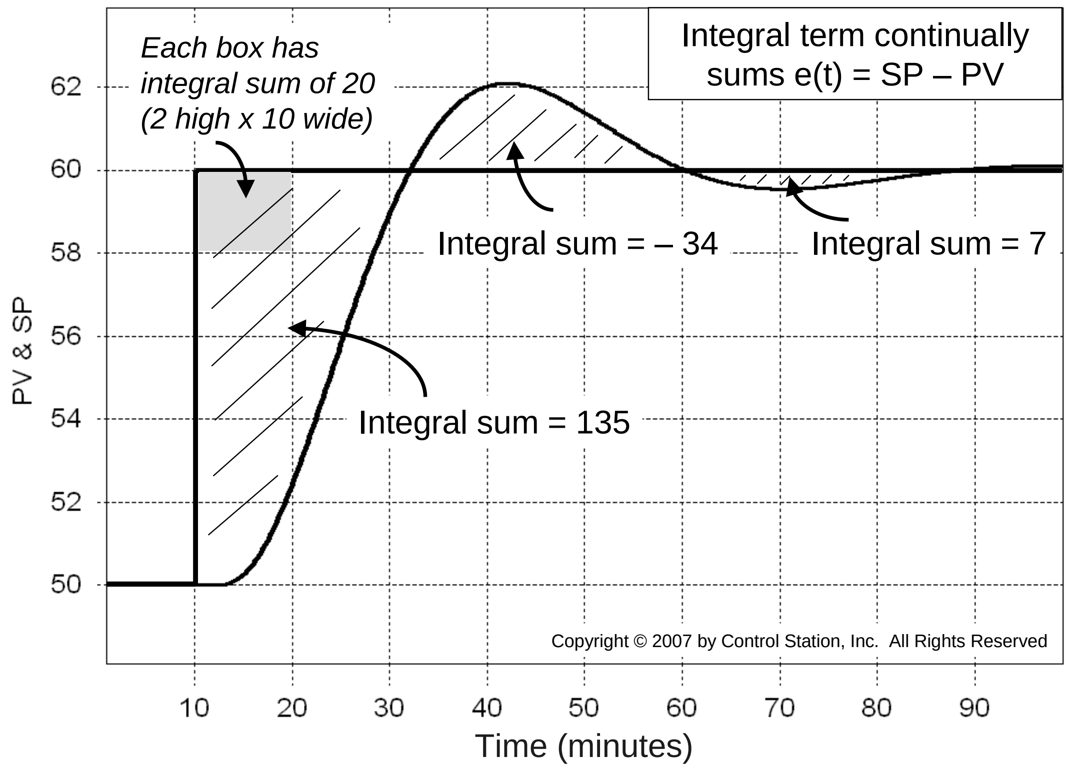
- The integral term continually sums up error, e(t)

- Through constant summing, integral action accumulates influence based on how long and how far the measured PV has been from SP over time.

- Even a small error, if it persists, will have a sum total that grows over time and the amount added to CO_{bias} will similarly grow.

- The continual summing of integration starts from the moment the controller is put in automatic

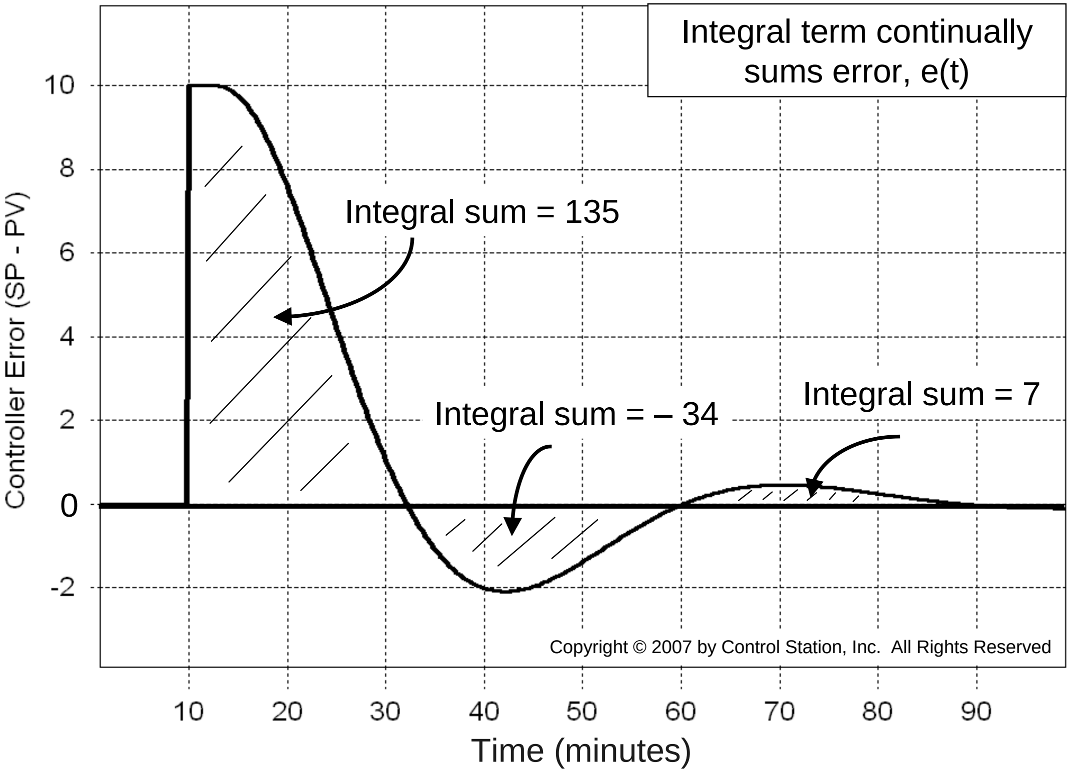
Integral Term Continually Sums the Value: SP – PV



- The integral is the sum of the area between SP and PV
- At t=32 min, when the PV first reaches the SP, the integral is:

$$\int_{0\,min}^{32min} e(t)dt = 135$$

Integral of Error is the Same as Integral of: SP – PV



- At t = 60 min, the total integral is: 135 – 34 = 101
- When the dynamics have ended, e(t) is constant at zero and the total integral has a final residual value: 135 – 34 + 7 = 108

Advantage of PI Control – No Offset

- The PI controller stops computing changes in CO when e(t) equals zero for a sustained period

$$CO = CO_{bias} + Kc \cdot e(t) + \frac{Kc}{\tau_I} \int e(t)dt$$

- At that point, the proportional term equals zero, and the integral term may have a residual value

$$CO = CO_{bias} + 0 + \underbrace{\frac{Kc}{\tau_I}(108)}$$

Integral acts as
"moving bias" term

- This residual value, when added to CO_{bias}, essentially creates an overall "moving bias" that tracks changes in operating level
- This moving bias eliminates offset, making PI control the **most widely used industry algorithm**

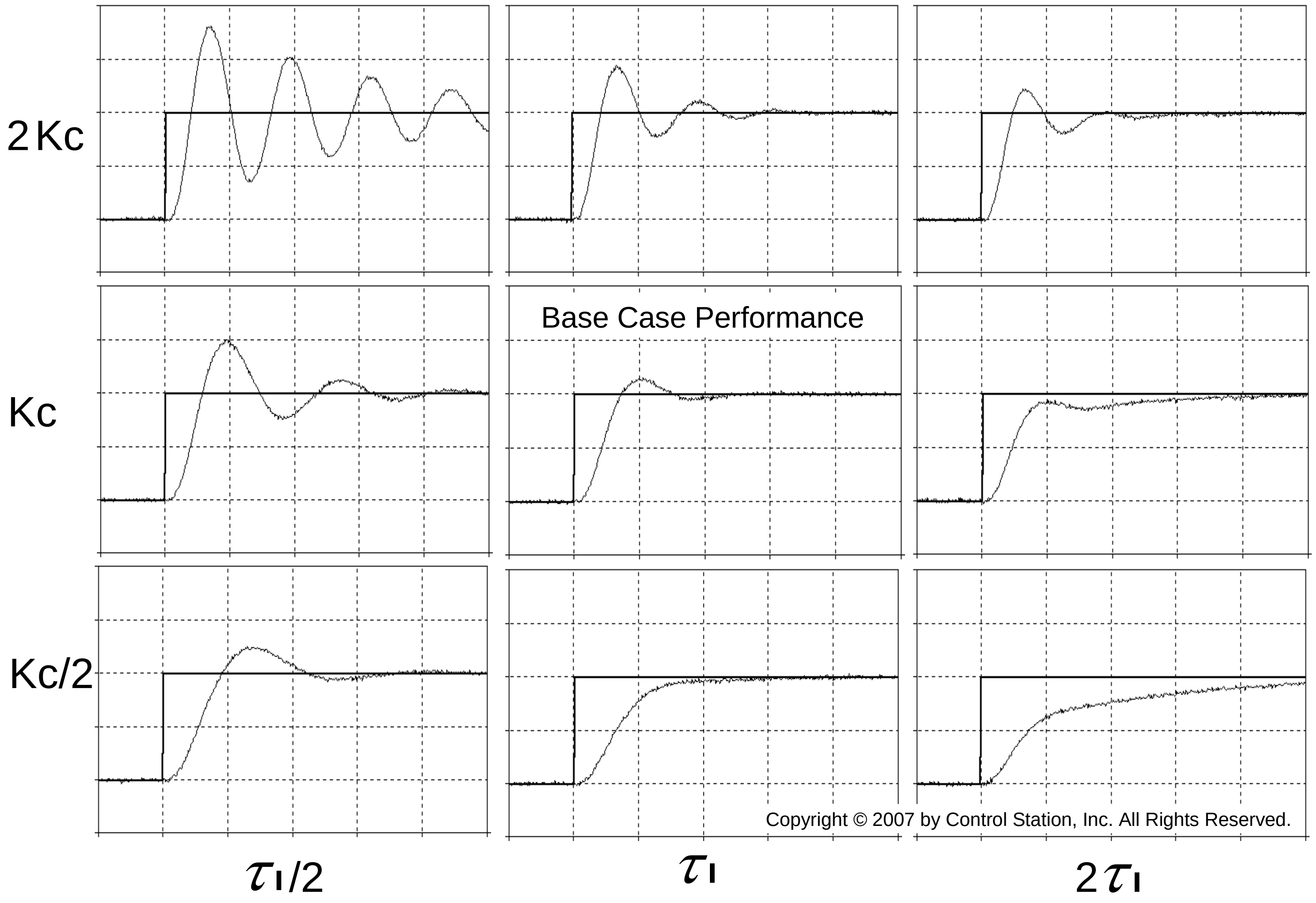
Disadvantages of PI Control - Interaction

- Integral action tends to increase the oscillatory or rolling behavior of the PV
- There are *two* tuning parameters (Kc and τ_I) and they interact with each other

$$CO = CO_{bias} + Kc \cdot e(t) + \frac{Kc}{\tau_I}\int e(t)dt$$

- This interaction can make it challenging to arrive at "best" tuning values

PI Controller Tuning Guide (Figure 8.9)



Integral Action and Reset Windup

- The math makes it possible for the error sum (the integral) to grow very large.

$$CO = CO_{bias} + Kc \cdot e(t) + \underbrace{\frac{Kc}{\tau_I} \int e(t)dt}_{\textit{integral}}$$

- The integral term can grow so large that the total CO signal stops making sense (it can be signaling for a valve to be open 120% or negative 15%)
- “Windup” is when the CO grows to exceed the valve limits because the integral has reached a huge positive/negative value
- It is associated with the integral term, so it is called *reset windup*
- The controller can’t regulate the process until the error changes sign and the integral term shrinks sufficiently so that the CO value again makes sense (moves between 0 – 100%).

Reset Windup and Jacketing Logic

- Industrial controllers employ jacketing logic to halt integration when the CO reaches a maximum or minimum value
- Beware if you program your own controller because reset windup is a trap that novices fall into time and again
- If two controllers trade off regulation of a single PV (e.g. select control; override control), jacketing logic must instruct the inactive controller to stop integrating. Otherwise, that controller’s integral term can wind up.

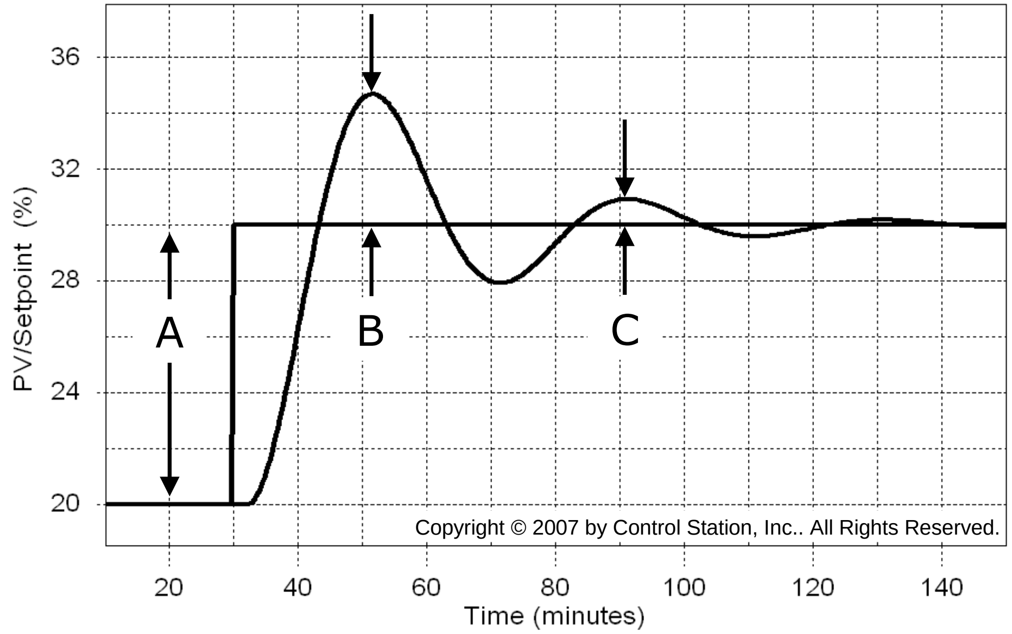
Evaluating Controller Performance

- Bioreactors can’t tolerate sudden operating changes because the fragile living cell cultures could die.
  - » “good” control means PV moves ***slowly***

- Packaging/filling stations can be unreliable. Upstream process must ramp back quickly if a container filling station goes down.
  - » “good” control means PV moves ***quickly***

- The operator or engineer defines what is good or best control performance based on their knowledge of:
  - goals of production
  - capabilities of the process
  - impact on down stream units
  - desires of management

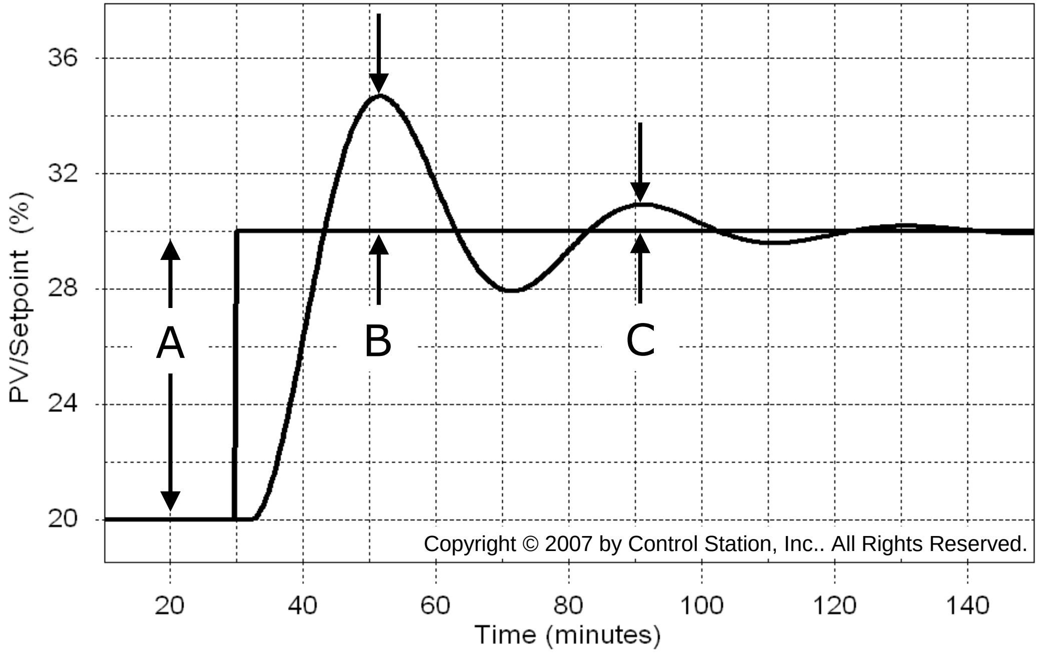
Performance Analysis



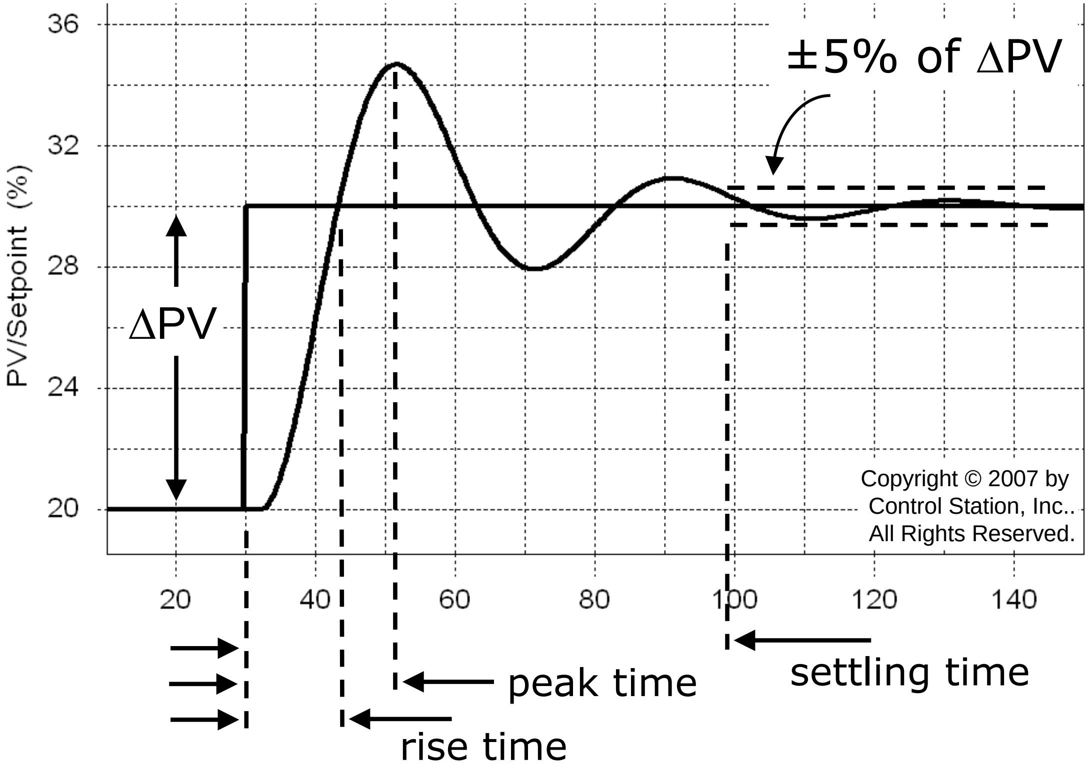
- Rise Time = When PV first reaches SP
- Peak Time = Time of first peak
- Overshoot Ratio = B/A
- Decay Ratio = C/B
- Settling Time = Time when PV remains < 5% of SP

Class Exercise

- Calculate:
  - Rise Time
  - Peak Time
  - Overshoot Ratio
  - Decay Ratio
  - Settling Time

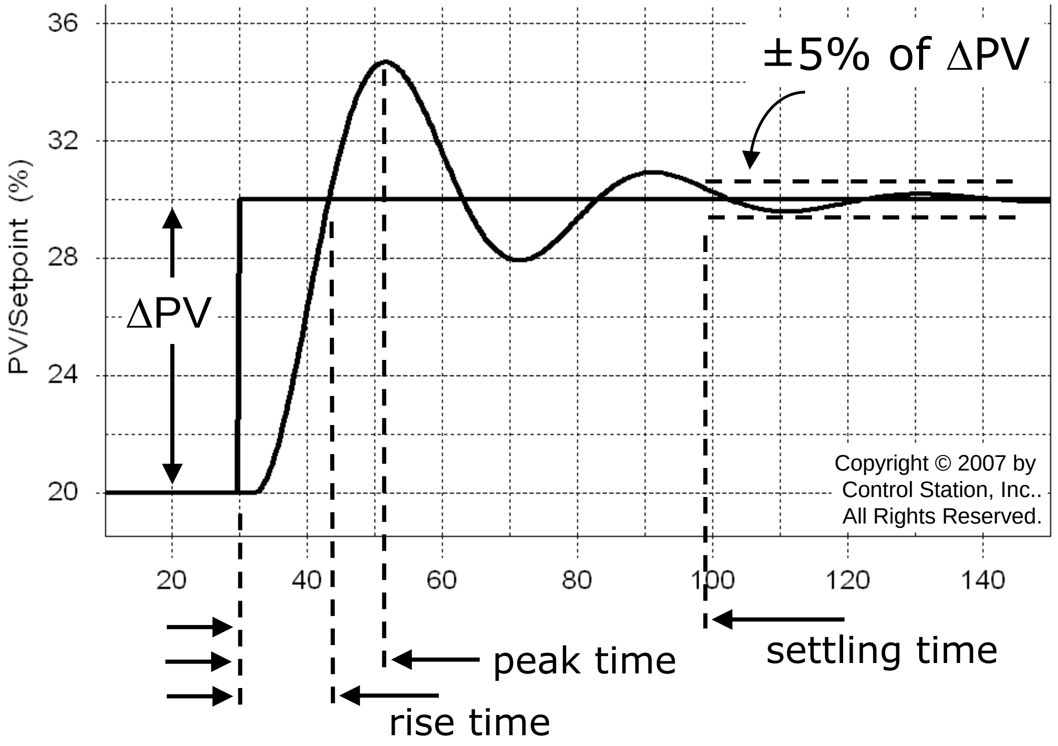


Performance Analysis - Time Related Criteria



- The clock for time related events begins when the SP is stepped

Performance Analysis - Time Related Criteria



- t_{rise} = 43 – 30 = 13 min
- t_{peak} = 51 – 30 = 13 min
- t_{settle} = 100 – 30 = 70 min

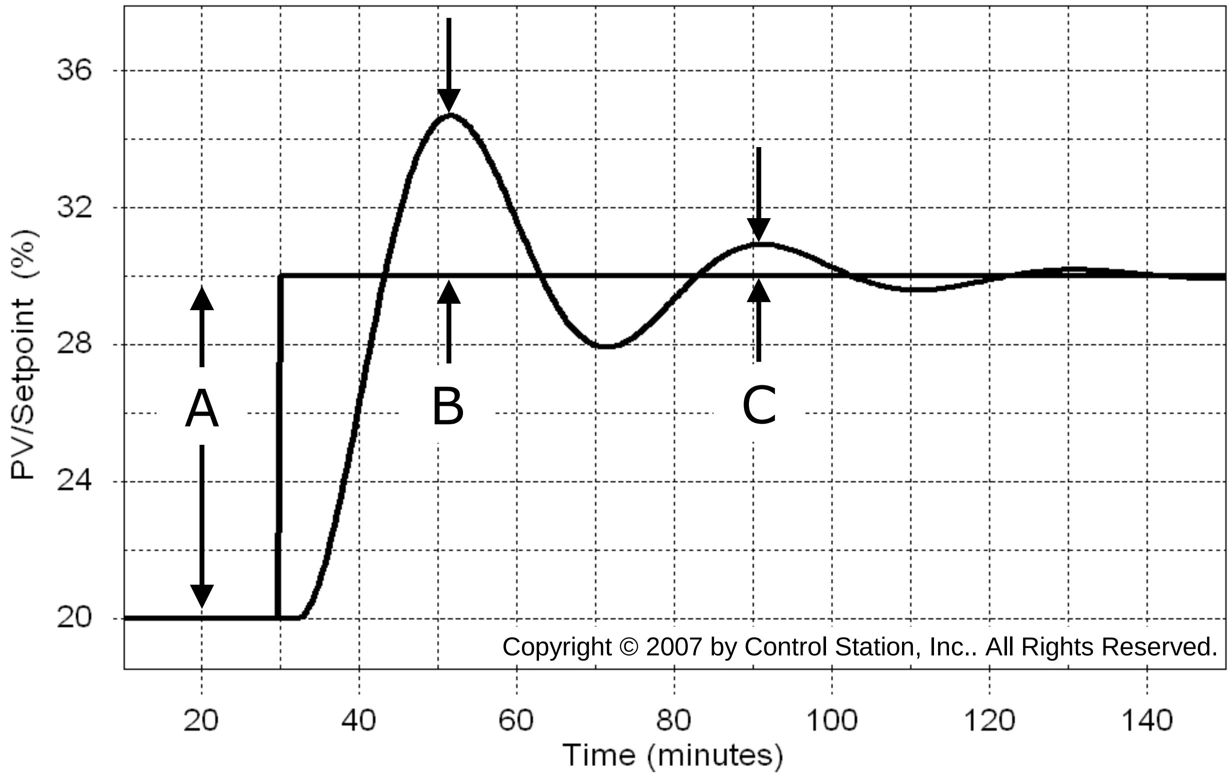
Performance Analysis - Peak Related Criteria

- A = (30 − 20)
  = 10%

- B = (34.5 − 30)
  = 4.5%

- C = (31 − 30)

  = 1%

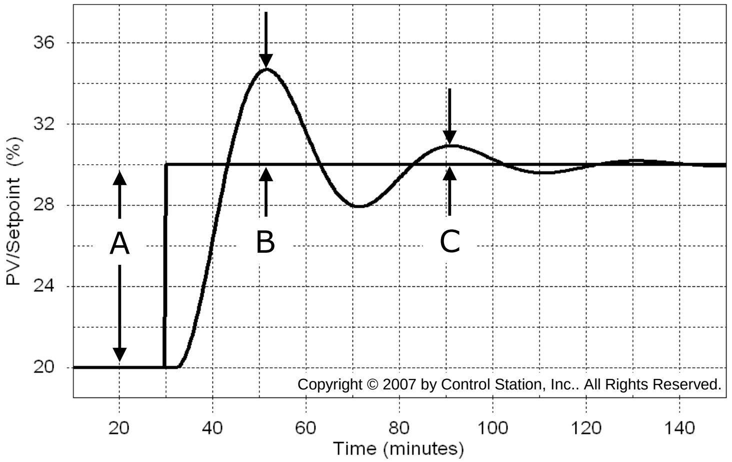


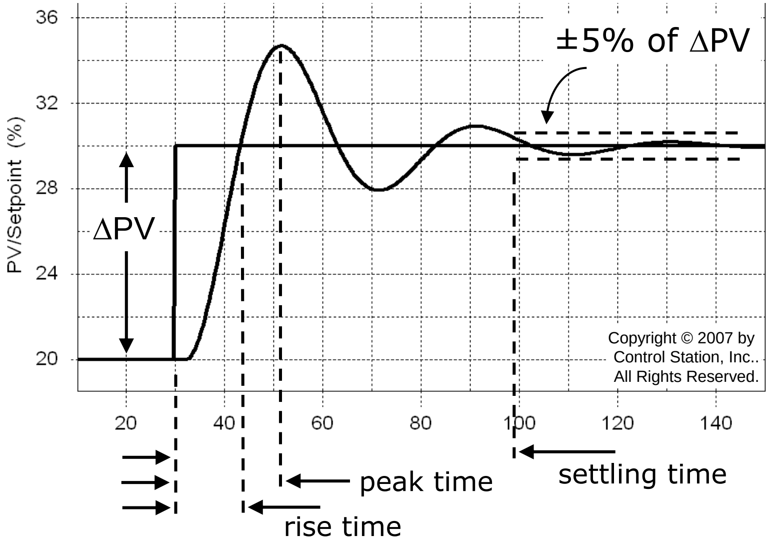
Here:

- Overshoot = 4.5/10 = 0.45 or 45%
- Decay ratio = 1/4.5 = 0.22 or 22%

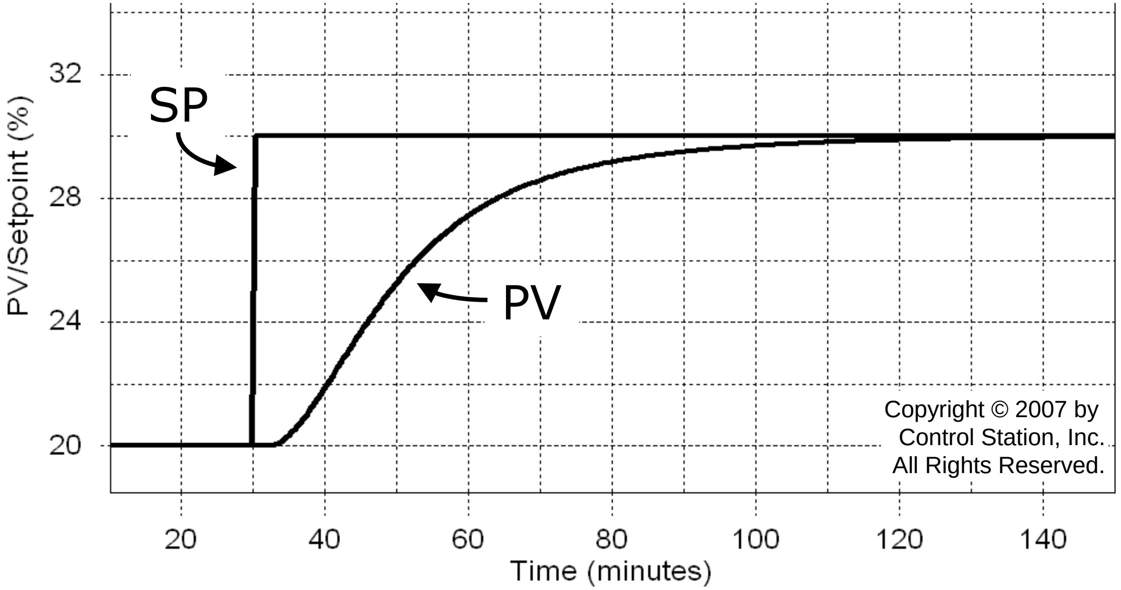
Performance Analysis Note

- The classical criteria are not independent:
  - if decay ratio is large, then likely will have a long settling time
  - if rise time is long, then likely will have a long peak time





Performance Analysis – What If No Peaks?



- Old rule-of-thumb is to design for a 10% Overshoot Ratio and/or a 25% decay ratio (called a quarter decay)
- Yet many modern operations want no PV overshoot at all, making B = C = 0
- With no peaks, the performance criteria are of limited value