Optimization

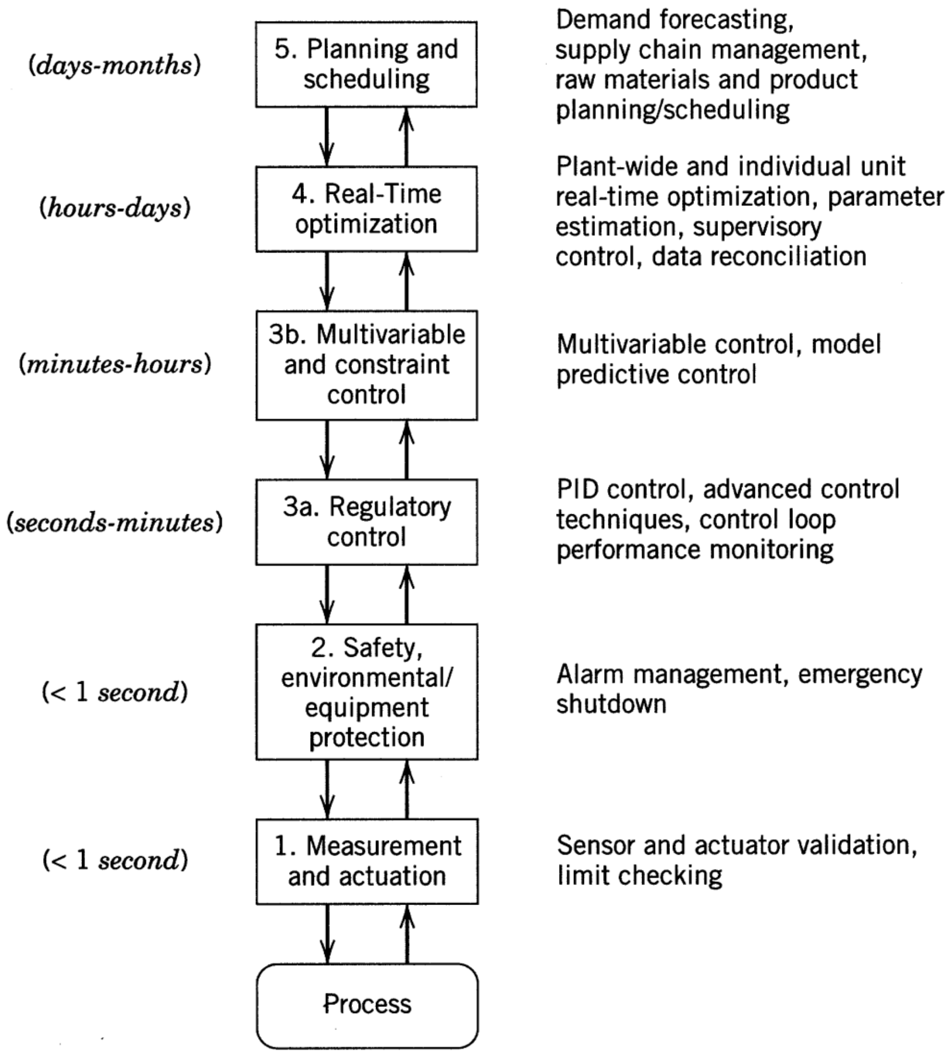


Figure 19.1 The five levels of process control and optimization in manufacturing. Time scales are shown for each level.

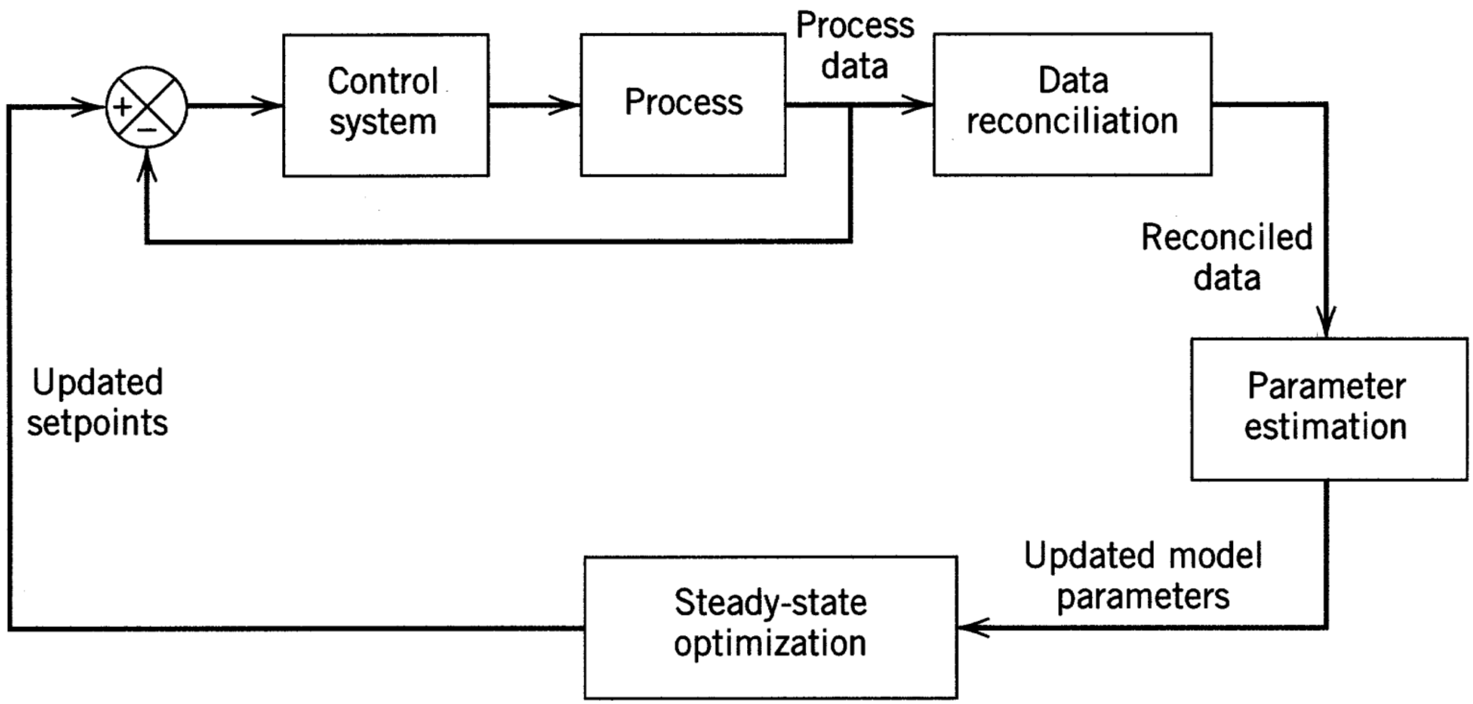


Figure 19.2 A block diagram for RTO and regulatory feedback control.

Constrained Optimization

- Optimization problems commonly involve equality and inequality constraints.
- Nonlinear Programming (NLP) Problems:
  - a) Involve nonlinear objective function (and possible nonlinear constraints).
  - b) Efficient off-line optimization methods are available (e.g. conjugate gradient, variable metric).
  - c) On-line use? May be limited by computer time and storage requirements.
- Quadratic Programming (QP) Problems:
  - a) Quadratic objective function plus linear equality and inequality constraints.
  - b) Computationally efficient methods are available.

- **Linear Programming (LP) Problems**
  Both objective function and constraints are linear.
  Solutions are highly structured and can be rapidly obtained.

Linear Programming (LP)

- Has gained widespread industrial acceptance for on-line optimization, blending etc.

- Linear constraints can arise due to:
  1. Production limitation e.g. equipment limitations, storage limits, market constraints.
  2. Raw material limitation
  3. Safety restrictions, e.g. allowable operating ranges for temperature and pressures.
  4. Physical property specifications e.g. product quality constraints when a blend property can be calculated as an average of pure component properties:

$$\bar{P} = \sum_{i=1}^{n} y_i P_i \leq \alpha$$

5. Material and Energy Balances
   - Tend to yield equality constraints.
   - Constraints can change frequently, e.g. daily or hourly.

• **Effect of Inequality Constraints**

- Consider the linear and quadratic objective functions on the next page.
- Note that for the LP problem, the optimum must lie on one or more constraints.

• **General Statement of the LP Problem:**

$$\max f = \sum_{i=1}^{n} c_i x_i$$

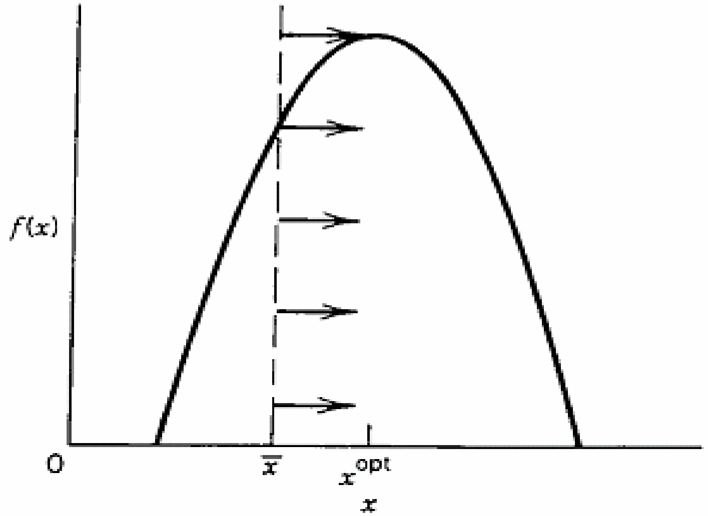
subject to:

$$x_i \geq 0 \qquad i = 1, 2, \ldots, n$$

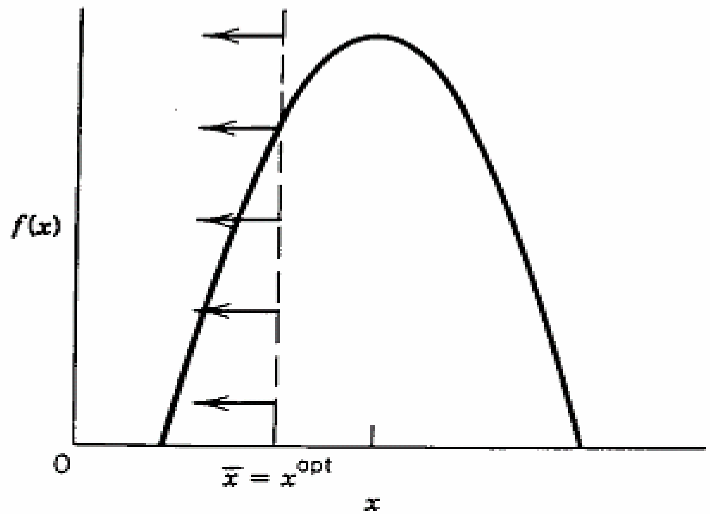
$$\sum_{j=1}^{n} a_{ij} x_j \leq b_i \qquad i = 1, 2, \ldots, n$$

• **Solution of LP Problems**

- Simplex Method
- Examine only constraint boundaries
- Very efficient, even for large problems

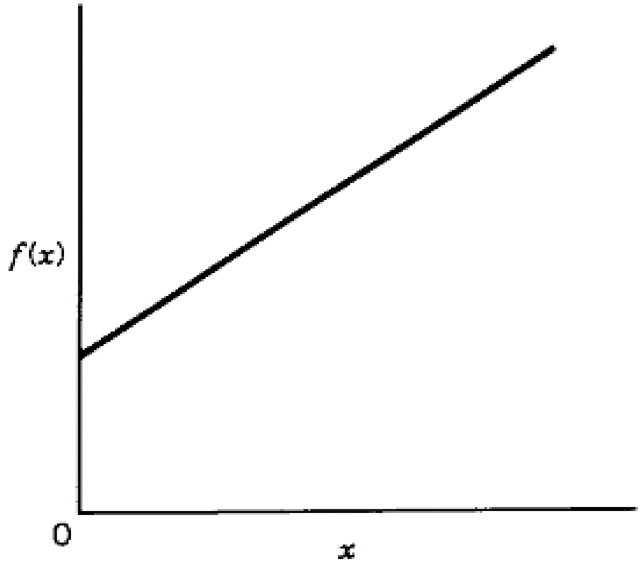


(a) Constrained case ($x \geq \bar{x}$), $x^{\text{opt}} = \dfrac{-a_1}{2a_2}$

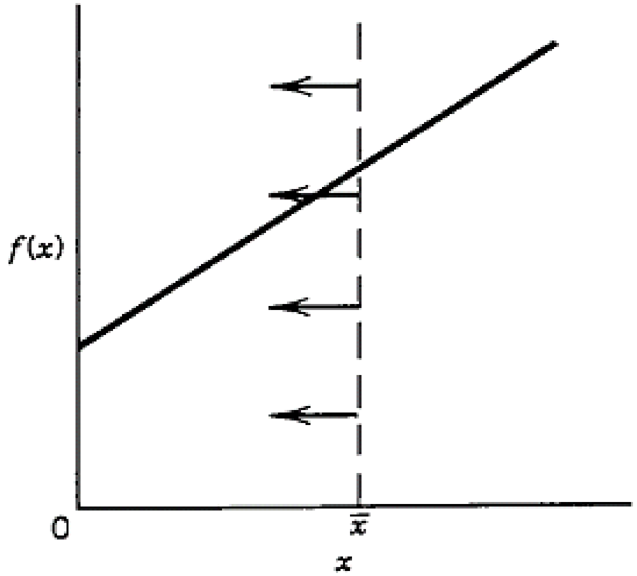


(b) Constrained case ($x \leq \bar{x}$), $x^{\text{opt}} = \bar{x}$

Figure The effect of an inequality constraint on the maximum of quadratic function, $f(x) = a_0 + a_1 x + a_2 x^2$ (The arrows indicate the allowable values of x.)



(a) Unconstrained case, $x^{\text{opt}} \rightarrow \infty$



(b) Constrained case ($x \leq \bar{x}$), $x^{\text{opt}} = \bar{x}$

The effect of a linear constraint on the maximum of linear objective function, $f(x) = a_0 + a_1 x$.

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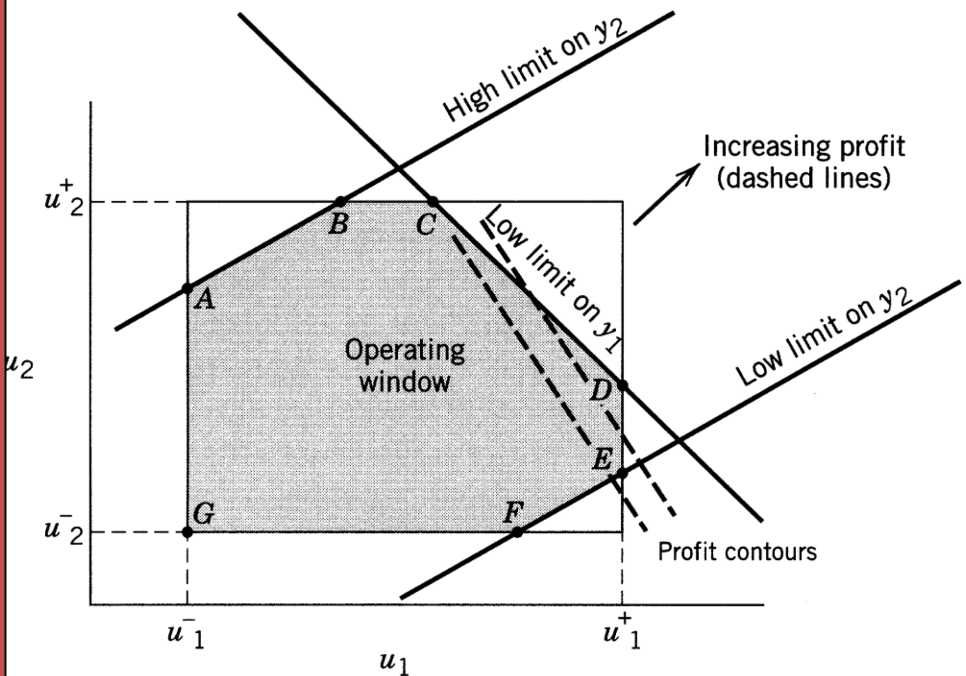


Figure 19.6 Operating window for a 2 x 2 optimization problem. The dashed lines are objective function contours, increasing from left to right. The maximum profit occurs where the profit line intersects the constraints at vertex *D*.

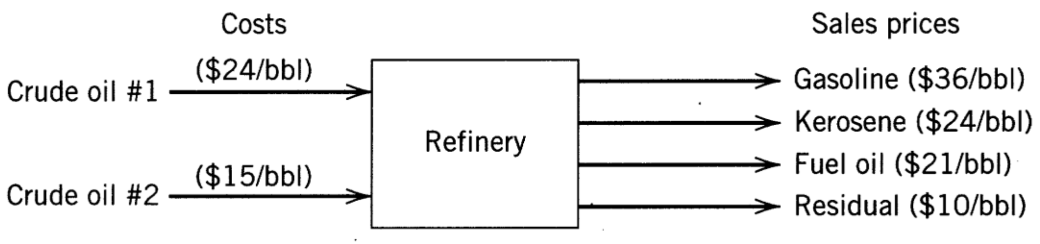
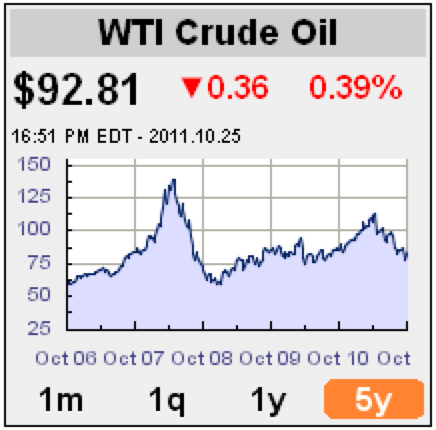
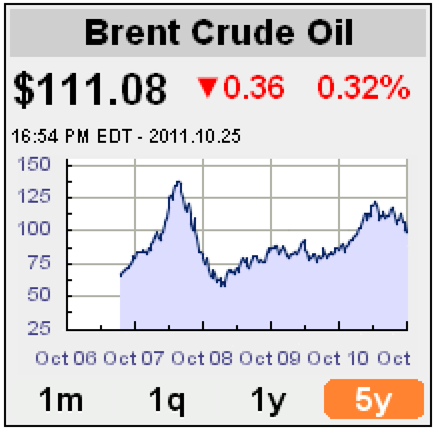


Figure 19.7 Refinery input and output schematic.





http://www.oil-price.net/

Table 19.3 Data for the Refinery Feeds and Products

	Volume percent yield		Maximum allowable production (bbl/day)
	Crude #1	Crude #2	
Gasoline	80	44	24,000
Kerosene	5	10	2,000
Fuel oil	10	36	6,000
Processing cost ($/bbl)	0.50	1.00	

Solution

Let x_1 = crude #1 (bbl/day)
x_2 = crude #2 (bbl/day)

Maximize profit (minimize cost):

y = income - raw mat'l cost - proc.cost

Calculate amounts of each product produced:

$$\begin{aligned}
\text{gasoline} &= 0.80\,x_1 + 0.44\,x_2\\
\text{kerosene} &= 0.05\,x_1 + 0.10\,x_2\\
\text{fuel oil} &= 0.10\,x_1 + 0.36\,x_2\\
\text{residual} &= 0.05\,x_1 + 0.10\,x_2
\end{aligned}$$

Income

gasoline	$(36)(0.80\,x_1 + 0.44\,x_2)$
kerosene	$(24)(0.05\,x_1 + 0.10\,x_2)$
fuel oil	$(21)(0.10\,x_1 + 0.36\,x_2)$
residual	$(10)(0.05\,x_1 + 0.10\,x_2)$

So,

Income $= 32.6\ x_1 + 26.8\ x_2$

Raw mat'l cost $= 24\ x_1 + 15\ x_2$

Processing cost $= 0.5\ x_1 + x_2$

Then, the objective function is

$$\text{Profit} = y = 8.1\ x_1 + 10.8\ x_2$$

<u>Constraints</u>

Maximum allowable production:

$0.80\ x_1 + 0.44\ x_2 \leq 24{,}000$ (gasoline)

$0.05\ x_1 + 0.10\ x_2 \leq 2{,}000$ (kerosene)

$0.10\ x_1 + 0.36\ x_2 \leq 6{,}000$ (fuel oil)

and, of course, $x_1 \geq 0, \quad x_2 \geq 0$

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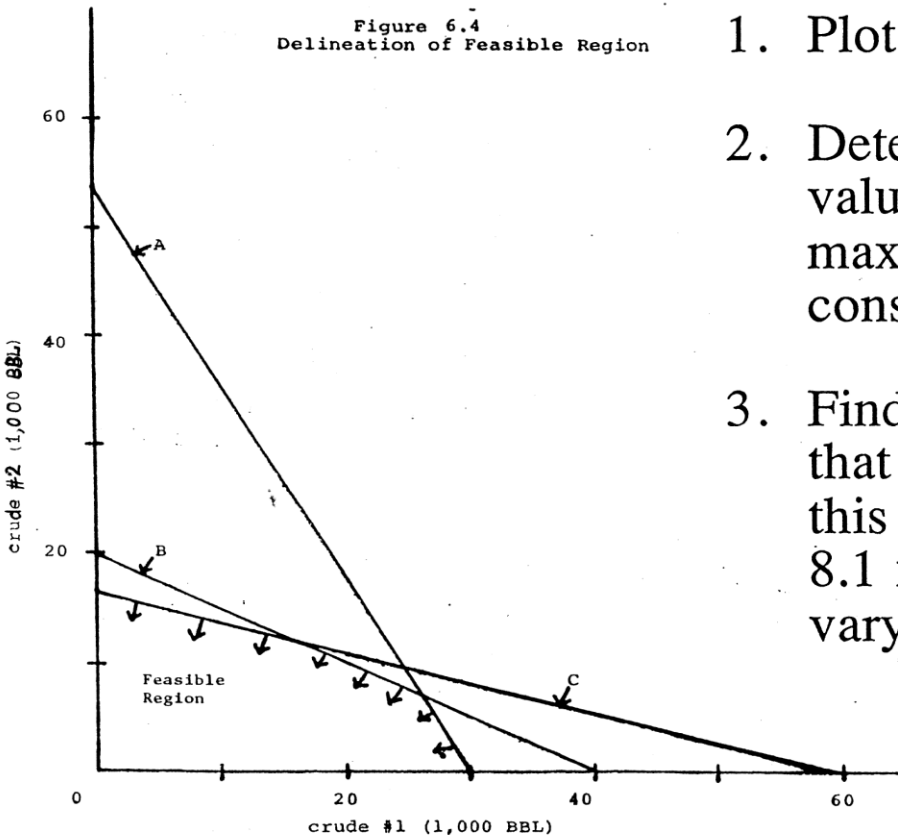
$$.80x_1 + .44x_2 \le 24{,}000 \quad (A)$$
$$.05x_1 + .10x_2 \le 2{,}000 \quad (B)$$
$$.10x_1 + .36x_2 \le 6{,}000 \quad (C)$$

Figure 6.4
Delineation of Feasible Region



Graphical Solution

1. Plot constraint lines on x_1-x_2 plane.

2. Determine feasible region (those values of x_1 and x_2 that satisfy maximum allowable production constraints.

3. Find point or points in feasible region that maximize $y = 8.1\ x_1 + 10.8\ x_2$; this can be found by plotting the line $8.1\ x_1 + 10.8\ x_2 = P$, where P can vary, showing different profit levels.

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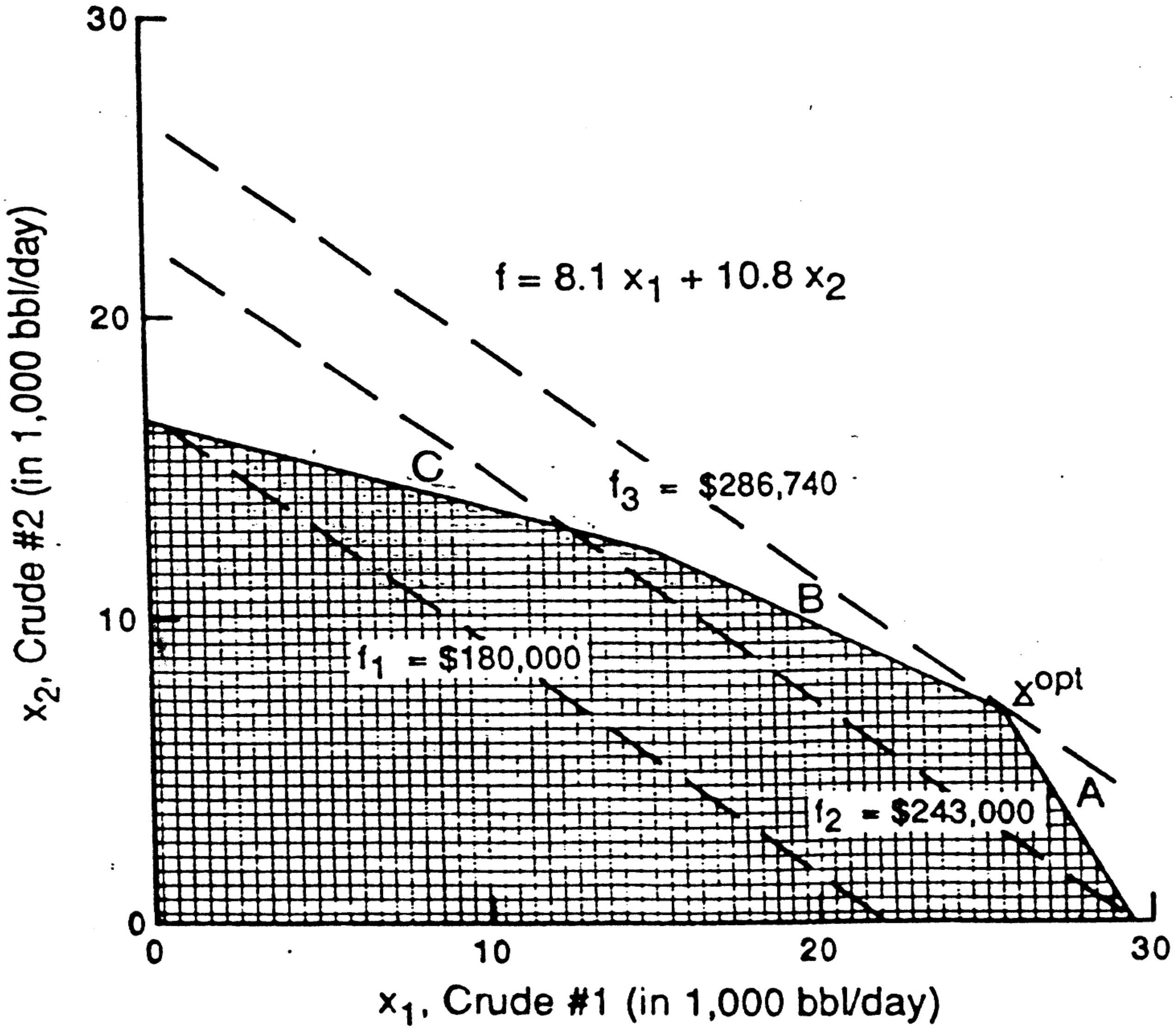


Figure 14

Feasible Region With Parameterization of Objective Function in Linear Programming

From the graph,

$x_1^{opt} \sim 26{,}000$
$x_2^{opt} \sim 7{,}000$

More precisely, this is the intersection of the first two constraints, so x_1^{opt} and x_2^{opt} can be solved for simultaneously:

$$0.80\ x1 + 0.44\ x2 = 34{,}000$$
$$0.50\ x1 + 0.10\ x2 = 2{,}000$$

$\Rightarrow\ x_1^{opt} = 26{,}200$ and $x_2^{opt} = 6{,}900$

with $P = \$\ 286{,}740/\text{day}$

As expected, optimum is at a corner of the feasible region.

Investigate the profit at the other corners:

(x_1,x_2)	Profit
(0,16667)	180,000
(15000,12500)	256,500
(30000,0)	243,000

Optimization in Industry

- 10,000-10,000,000+ Variables Typical
- Need a general computational approach
- Numerical Methods

Optimization Tools

- Excel Solver
- AMPL
- APMonitor
- GAMS
- PIMMS
- Romeo
- etc…

Numerical Methods for Optimization

http://apmonitor.com/online/view_pass.php?f=crude_oil.apm

Click the green arrow to solve

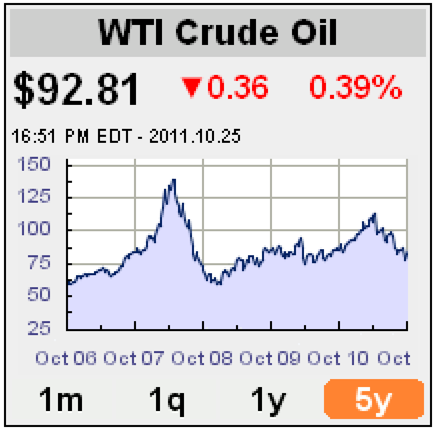
View the solution by clicking on the solution table

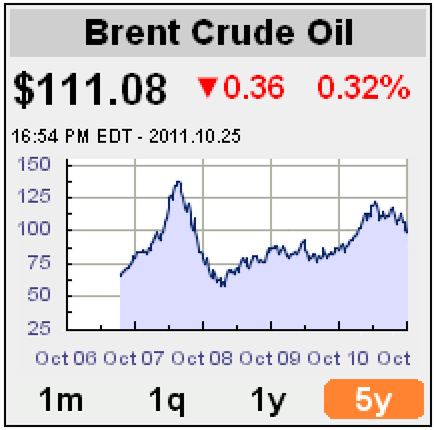
```
Model
  Variables
    crude[1:2] >= 0
    gasoline   >= 0, <= 24000
    kerosene   >= 0, <= 2000
    fuel_oil   >= 0, <= 6000
    residual   >= 0
    income
    raw_matl_cost
    proc_cost
    profit
  End Variables

  Equations
    gasoline = 0.80 * crude[1] + 0.44 * crude[2]
    kerosene = 0.05 * crude[1] + 0.10 * crude[2]
```

Name	Lower	Value	Upper
ss.crude[1]	0.0000E+00	2.6207E+04	---
ss.crude[2]	0.0000E+00	6.8966E+03	---
ss.gasoline	0.0000E+00	2.4000E+04	2.4000E+04
ss.kerosene	0.0000E+00	2.0000E+03	2.0000E+03
ss.fuel_oil	0.0000E+00	5.1034E+03	6.0000E+03
ss.residual	0.0000E+00	2.0000E+03	---
ss.income	---	1.0392E+06	---
ss.raw_matl_cost	---	7.3241E+05	---
ss.proc_cost	---	2.0000E+04	---
ss.profit	---	2.8676E+05	---

Special Problem 11

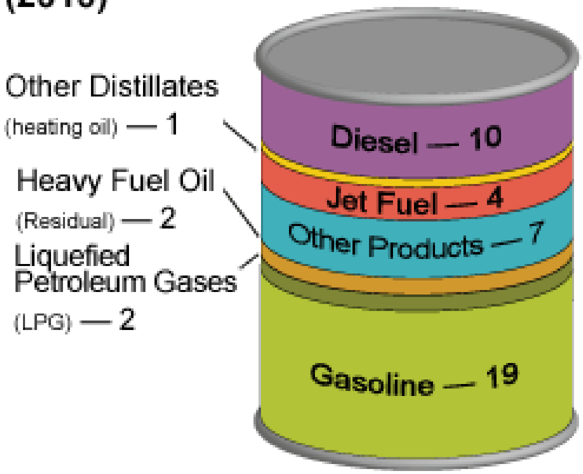




http://www.oil-price.net/

http://en.wikipedia.org/wiki/List_of_crude_oil_products

Products Made from a Barrel of Crude Oil (Gallons) (2010)



http://www.eia.gov/energyexplained/index.cfm?page=oil_refining

Optimization Model in APMonitor

```
Model
  Variables
    crude[1:2] >= 0
    gasoline   >= 0, <= 24000
    kerosene   >= 0, <= 2000
    fuel_oil   >= 0, <= 6000
    residual   >= 0
    income
    raw_matl_cost
    proc_cost
    profit
  End Variables

  Equations
    gasoline = 0.80 * crude[1] + 0.44 * crude[2]
    kerosene = 0.05 * crude[1] + 0.10 * crude[2]
    fuel_oil = 0.10 * crude[1] + 0.36 * crude[2]
    residual = 0.05 * crude[1] + 0.10 * crude[2]

    income = 36 * gasoline + 24 * kerosene + 21 * fuel_oil + 10 * residual

    raw_matl_cost = 24 * crude[1] + 15 * crude[2]

    proc_cost = 0.5 * crude[1] + 1.0 * crude[2]

    profit = income - raw_matl_cost - proc_cost

    maximize profit
  End Equations
End Model
```