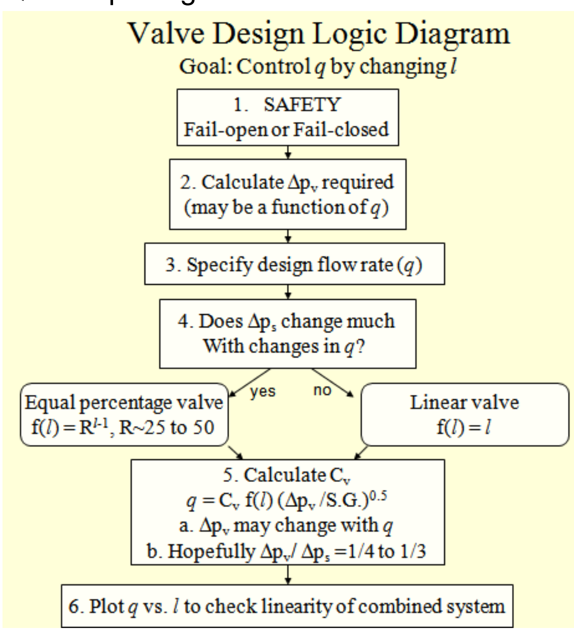
ChE436 - Process Control
Final Exam Review Sheet

<u>Vocabulary</u>

- Process Variable (PV)
- Set Point (SP)
- Controller Output (OP)
- Manipulated Variable (MV)
- Disturbances (D)
- Tests to obtain empirical models
    - Step test
    - Impulse test
    - Doublet test
    - Pseudo-random binary sequence (PRBS)
    - etc...
- Valves
    - Linear
    - Equal Percentage
    - Quick Opening



- Architectures for improved disturbance rejection
    - Feed Forward
    - Cascade

<u>Concepts</u>

- Linear vs. Nonlinear Systems
- For First Order Systems
    - Gain
    - Time Constant
    - Dead-Time
- For Second Order Systems

    - Rise Time
    - Settling Time
    - Damping Ratio
    - Peak Time
- To obtain good data for tuning, the controller output must force the process variable to move at least 10 times the noise band (signal to noise ratio >= 10)
- PID Controller Options
    - P-only
        - Accelerates the response of controlled process
        - Produces offset except for integrating (1/s) processes
    - PI
        - Most commonly used in industrial practice
        - Eliminates offset
        - Usually higher maximum deviations than P-only
        - Poor tuning leads to sluggish, long oscillating responses
        - Increased gain may lead to larger oscillations and instability
    - PID
        - Introduces stabilizing effect on closed-loop response
        - Exacerbates noise
        - May cause additional wear on valves, etc.
- Partial Fractions

<u>Definitions</u>

1. Process Gain

$$K_P = \frac{\text{Steady State Change in the Measured Process Variable, } \Delta y(t)}{\text{Steady State Change in the Controller Output, } \Delta u(t)}$$

2. Degrees of Freedom (DOF)

$$N_{DOF} = N_{Variables} - N_{Equations}$$

3. Laplace Transform

$$F(s) = \mathcal{L}\left[f(t)\right] = \int_0^\infty f(t) e^{-st} dt$$

a. Laplace Transform of a constant

$$\mathcal{L}(a) = \int_0^\infty a e^{-st} dt = -\frac{a}{s} e^{-st} \Big|_0^\infty = 0 - \left(-\frac{a}{s}\right) = \boxed{\frac{a}{s}}$$

b. Laplace Transform of a derivative

$$\mathcal{L}\left[\frac{df}{dt}\right] = sF(s) - f(0)$$

c. Note: Complete Laplace Transform Table will be Attached

d. Note: Laplace Transform Table also available on pg. 42-43 of SEMD

7. Characteristic Equation

- 1+G_OL = 0
- One positive root (real part) indicates an unstable system
- Imaginary roots indicates oscillations in response
- Quadratic formula to determine roots $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

- Example:

$$\begin{bmatrix} 2+6i \\ 2-6i \\ -1 \\ -3 \\ -2 \end{bmatrix} \begin{matrix} \text{Oscillatory, diverges} \\ \text{Oscillatory, diverges} \\ \text{No oscillations, converges} \\ \text{No oscillations, converges} \\ \text{No oscillations, converges} \end{matrix}$$

Overall: Oscillatory, diverges

8. Dead-time Approximations

a) Taylor Series Approximation

$$e^{-\theta_0 s} \approx 1 - \theta_0 s$$

$$e^{-\theta_0 s} = \frac{1}{e^{\theta_0 s}} \approx \frac{1}{1+\theta_0 s}$$

b) Pade Approximation

$$e^{-\theta s} \approx \frac{1-\frac{\theta}{2}s}{1+\frac{\theta}{2}s}$$

c) Skogestad's method for approximating higher order systems with FOPDT

i) Largest time constant becomes tau_p

ii) Second largest time constant is split between theta_p and tau_p

iii) Other time constants lumped into theta_p

Transient balance equations

$$\frac{\begin{bmatrix}\text{accumulation of } S \\ \text{within a system}\end{bmatrix}}{\text{time period}} = \frac{\begin{bmatrix}\text{flow of } S \\ \text{into the system}\end{bmatrix}}{\text{time period}} - \frac{\begin{bmatrix}\text{flow of } S \\ \text{out of the system}\end{bmatrix}}{\text{time period}} + \frac{\begin{bmatrix}\text{amount of } S \\ \text{generated within} \\ \text{the system}\end{bmatrix}}{\text{time period}} - \frac{\begin{bmatrix}\text{amount of } S \\ \text{consumed within} \\ \text{the system}\end{bmatrix}}{\text{time period}}$$

Accumulation = In - Out + Generation - Consumption

Overall Mass Balance

$$\frac{dm}{dt} = \frac{d(\rho V)}{dt} = \sum_{i=inlet} \dot{m}_i - \sum_{j=outlet} \dot{m}_j$$

Species Balance for Each Component

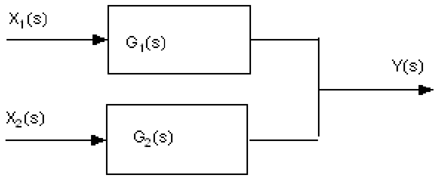
$$\frac{dn_A}{dt} = \frac{d(c_A V)}{dt} = \sum_{i=inlet} c_{Ai} q_i - \sum_{j=outlet} c_{Aj} q_j + r_A V$$

Energy Balance

$$\frac{d[\rho C_p V(T - T_{ref})]}{dt} = \sum_{i:inlet} w_i C_p (T_i - T_{ref}) - \sum_{j:outlet} w_j C_p (T_j - T_{ref}) + Q + W_s$$

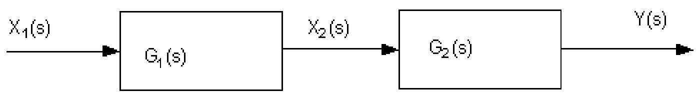
Forms of basic transfer functions

Transfer functions in parallel



Y(s) = X_1(s) G_1(s) + X_2(s) G_2(s)

Transfer functions in series



Y(s) = X_2(s) G_2(s)
X_2(s) = X_1(s) G_1(s)
So Y(s) = X_1(s)G_1(s) G_2(s)

<u>First Order Systems</u>

$$\tau_p \frac{\partial x}{\partial t} = -x + K_p u(t - \theta_p)$$

$$\tau_p = \text{Process time constant}$$

$$K_p = \text{Process Gain}$$

$$\theta_p = \text{Process dead-time}$$

Why is tau = 63.2% to steady-state?

$$\tau \frac{\partial y}{\partial t} = -y + Ku \quad \text{drop the time-delay from FOPDT equation}$$

$$\tau s Y(s) - y(0) = -Y(s) + KU(s) \quad \text{LaPlace Transform (pg. 42 of SEMD)}$$

$$\frac{Y(s)}{U(s)} = \frac{K}{\tau s + 1} \quad \text{Rearrange with y(0) = 0 to obtain Transfer Function}$$

$$Y(s) = \frac{1}{s}\left(\frac{K}{\tau s + 1}\right) \quad \text{U(s) = 1/s (step function)}$$

$$y(t) = K\left(1 - e^{-\frac{t}{\tau}}\right) \quad \text{Inverse LaPlace Transform (pg.42 of SEMD)}$$

$$y(t) = K(1 - e^{-1}) = K(0.632) \quad \text{At } t = \tau$$

- t = 1 tau = 0.63
- t = 2 tau = 0.86
- t = 3 tau = 0.95
- t = 4 tau = 0.98
- t = 5 tau = 0.9933

Graphical method for obtaining FOPDT model

1. Find θ_p
2. Find y_∞
3. Find Δy_{max}
4. Find $y_{0.632}$
5. Find $t_{0.632}$
6. Find τ_p
7. Find $K_p = \Delta y_{max} / \Delta u$

Can also obtain FOPDT from:

- Least squares estimation of Kp, tau_p, theta_p
- Linearization of first principles model

Second Order Systems

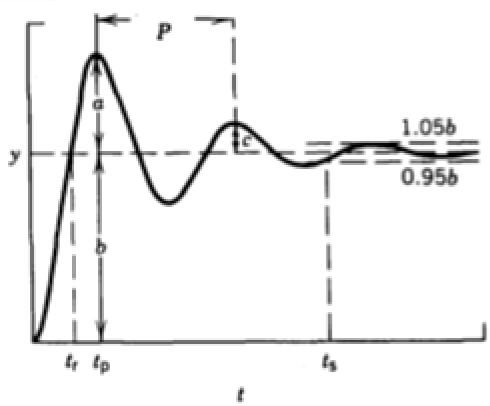
$$G(s) = \frac{K}{\tau^2 s^2 + 2\zeta\tau s + 1}$$

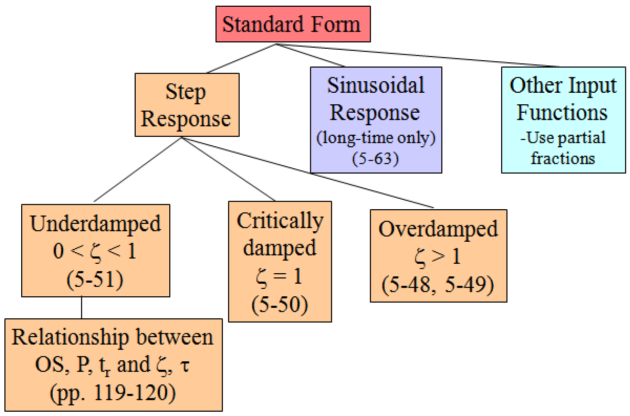
K = Gain
τ = Natural Period of Oscillation
ζ = Damping Factor (zeta)

$\zeta > 1$	Overdamped	Two distinct real roots
$\zeta = 1$	Critically Damped	Two equal real roots
$0 < \zeta < 1$	Underdamped	Two complex conjugate roots

Overdamped Eq. 5-48 or 5-49	Sluggish, no oscillations
Critically damped Eq. 5-50	Faster than overdamped, no oscillation
Underdamped Eq. 5-51	Fast, oscillations occur

1. **Rise Time:** t_r is the time the process output takes to first reach the new steady-state value.
2. **Time to First Peak:** t_p is the time required for the output to reach its first maximum value.
3. **Settling Time:** t_s is defined as the time required for the process output to reach and remain inside a band whose width is equal to ±5% of the total change in y. The term 95% response time sometimes is used to refer to this case. Also, values of ±1% sometimes are used.
4. **Overshoot:** $OS = a/b$ (% overshoot is $100a/b$).
5. **Decay Ratio:** $DR = c/a$ (where c is the height of the second peak).
6. **Period of Oscillation:** P is the time between two successive peaks or two successive valleys of the response.





Second order plus dead-time (overdamped system)

$$\tau_{P1}\tau_{P2}\frac{d^2y(t)}{dt^2}+(\tau_{P1}+\tau_{P2})\frac{dy(t)}{dt}+y(t)=K_P\,u(t-\theta_P)$$

<u>Proportional Integral Derivative (PID) Controllers</u>

Controller in Time Domain (Derivative on Measurement)

$$OP=OP_{bias}+K_c e(t)+\frac{K_c}{\tau_I}\int e(t)dt-K_c\tau_D\frac{\partial PV}{\partial t}$$

where:

- OP = controller output signal (also seen as CO in PPC)
- OP_{bias} = controller bias or null value
- PV = measured process variable
- SP = set point
- e(t) = controller error = SP – PV
- Kc = controller gain (a tuning parameter)
- τ_I = controller reset time (a tuning parameter)
- τ_D = controller derivative action (a tuning parameter)

Controller in Laplace Domain (Derivative on Error)

$$\frac{P'(s)}{E(s)}=K_c\left[1+\frac{1}{\tau_I s}+\tau_D s\right]$$

<u>PID Controller Tuning</u>

PID Tuning Guide

Begin by fitting a first order plus dead time (FOPDT) dynamic model to process data. "Process" is defined to include all dynamic information from the output signal of the controller through the measured response signal of the process variable.

Generate process data by forcing the measured process variable with a change in the controller output signal. For accurate results:

- the process must begin at steady state; the first data point recorded to file must equal that steady state value
- the data collection sample rate should be ten times per time constant or faster ($T \le 0.1\,\tau_P$)
- the controller output should force the measured process variable to move at least ten times the noise band

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Use *Design Tools* to fit a FOPDT dynamic model to the process data set. A FOPDT model has the form:

Time Domain: $\tau_P \dfrac{dy(t)}{dt} + y(t) = K_P\, u(t - \theta_P)$

where:
- $y(t)$ = measured process variable signal
- $u(t)$ = controller output signal
- K_P = process gain; units of $y(t)/u(t)$
- τ_P = process time constant; units of time
- θ_P = process dead time; units of time

Laplace Domain: $\dfrac{Y(s)}{U(s)} = \dfrac{K_P\, e^{-\theta_P s}}{\tau_P s + 1}$

also:
- K_C = controller gain; units of $u(t)/y(t)$
- τ_I = controller reset time; units of time
- τ_D = controller derivative time; units of time
- α = derivative filter constant; unitless

Values of K_P, τ_P and θ_P that describe the dynamic behavior of your process are important because:

- they are used in correlations (listed below) to compute initial PID controller tuning values K_C, τ_I, τ_D and α
- the sign of K_P indicates the action of the controller ($+K_P \rightarrow$ reverse acting; $-K_P \rightarrow$ direct acting)
- the size of τ_P indicates the maximum desirable loop sample time (be sure sample time $T \le 0.1\,\tau_P$)
- the ratio θ_P/τ_P indicates whether a Smith predictor would show benefit (useful when $\theta_P/\tau_P > 0.7$)
- the model itself is used in feed forward, Smith predictor, decoupling and other model-based controllers

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These correlations provide a starting point for tuning. Final tuning requires online trial and error. "Best" tuning is defined by you and your knowledge of the capabilities of the process, desires of management, goals of production, and impact on other processes.

IMC (lambda) Tuning

Standard Tuning: τ_C is the larger of $0.1\tau_P$ or $0.8\theta_P$

Conservative Tuning: τ_C is the larger of $0.5\tau_P$ or $4.0\theta_P$

* This is an ITAE correlation as no P-Only IMC exists

	K_C	τ_I	τ_D	α
P-Only*	$\dfrac{0.202}{K_P}(\theta_P/\tau_P)^{-1.219}$			
PI	$\dfrac{1}{K_P}\dfrac{\tau_P}{(\theta_P+\tau_C)}$	τ_P		
PID Ideal	$\dfrac{1}{K_P}\left(\dfrac{\tau_P+0.5\theta_P}{\tau_C+0.5\theta_P}\right)$	$\tau_P+0.5\theta_P$	$\dfrac{\tau_P\theta_P}{2\tau_P+\theta_P}$	
PID Interacting	$\dfrac{1}{K_P}\left(\dfrac{\tau_P}{\tau_C+0.5\theta_P}\right)$	τ_P	$0.5\theta_P$	
PID Ideal w/filter	$\dfrac{1}{K_P}\left(\dfrac{\tau_P+0.5\theta_P}{\tau_C+\theta_P}\right)$	$\tau_P+0.5\theta_P$	$\dfrac{\tau_P\theta_P}{2\tau_P+\theta_P}$	$\dfrac{\tau_C(\tau_P+0.5\theta_P)}{\tau_P(\tau_C+\theta_P)}$
PID Interacting w/filter	$\dfrac{1}{K_P}\left(\dfrac{\tau_P}{\tau_C+\theta_P}\right)$	τ_P	$0.5\theta_P$	$\dfrac{\tau_C}{\tau_C+\theta_P}$

<u>Linearization and deviation variables</u>

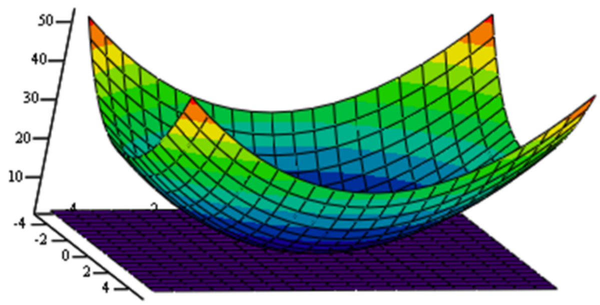
Procedure for obtaining transfer function from first principles (material and energy balances)

Derive → Linearize (Deviation Variables) → Laplace → Input Function → Solve

Linearization

$$f(x,y) := x^2 + y^2 + 2$$

$$f_{lin}(x,y) := f(x_{lin}, y_{lin}) + \left(\frac{d}{dx_{lin}} f(x_{lin}, y_{lin})\right)\cdot(x - x_{lin}) + \left(\frac{d}{dy_{lin}} f(x_{lin}, y_{lin})\right)\cdot(y - y_{lin})$$



Deviation Variables

$$x' = x - \bar{x}$$

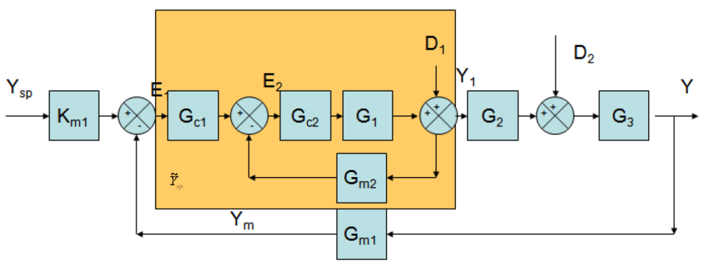
x' = Deviation Variable

x = Original Variable

$\bar{x}$ = Nominal Value

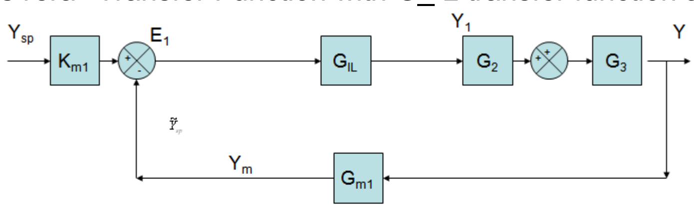
<u>Block diagram algebra</u>

- Inner Loop First with Shortcut Method (Y1/E = Direct / (1+Loop))



$$\frac{Y_1}{E} = G_{IL} = \frac{direct}{1+loop} = \frac{G_{c1}G_{c2}G_1}{1+G_{m2}G_{c2}G_1}$$

- Overall Transfer Function with G_IL transfer function and Shortcut Method



$$\frac{Y}{Y_{sp}} = \frac{K_{m1}G_{IL}G_2G_3}{1+G_{m1}G_{IL}G_2G_3}$$

<u>Transfer functions</u>

$$G(s) = \frac{Y(s))}{U(s))}$$

<u>Initial and final values from transfer functions</u>

- Final Value Theorem

$$y(\infty) = \lim_{s \to 0}[sY(s)]$$

- Initial Value Theorem

$$y(0) = \lim_{s \to \infty}[sY(s)]$$

- Controller Offset

$$offset = \lim_{s->0}(s(Y_{sp}(s) - Y(s)))$$

- Gain

$$K_p = \lim_{s \to 0} G(s)$$

<u>Write input function in Laplace coordinates from graph in time coordinates</u>

- Common Functions

Step	$u(s) = \frac{M}{s}$	(5-6)
Ramp	$u(s) = \frac{a}{s^2}$	(5-8)
Rectangular pulse	$u(s) = \frac{h}{s}\left(1 - e^{-t_w s}\right)$	(5-11)
Triangular pulse	$u(s) = \frac{2}{t_w}\left(\frac{1 - 2e^{-t_w s/2} + e^{-t_w s}}{s^2}\right)$	(5-13)
Sine wave	$u(s) = \frac{A\omega}{s^2 + \omega^2}$	(5-15)
Impulse	$u(s) = a$	p. 76

- Time Delay (function becomes non-zero after theta time)

In time domain:

- Replace t with (t-θ) and multiply by S(t-θ)

$$f(t-\theta) \cdot S(t-\theta)$$

In Laplace domain

- Multiply by $e^{-\theta s}$

$$e^{-\theta s}F(s)$$

<u>Stability analysis (Routh, Direct Substitution, Root Locus, Bode Plot)</u>

- Routh Array

$$a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0 = 0 \qquad (11\text{-}93)$$

$a_n > 0$ Multiply polynomial by -1 if a_n<0

Row				
1	a_n	a_{n-2}	a_{n-4}	$\cdots$
2	a_{n-1}	a_{n-3}	a_{n-5}	$\cdots$
3	b_1	b_2	b_3	$\cdots$
4	c_1	c_2	$\cdots$	
$\vdots$	$\vdots$			
$n+1$	z_1			

Stable if leading edge is positive

$$b_1 = \frac{a_{n-1}a_{n-2} - a_n a_{n-3}}{a_{n-1}} \qquad (11\text{-}94)$$

$$b_2 = \frac{a_{n-1}a_{n-4} - a_n a_{n-5}}{a_{n-1}} \qquad (11\text{-}95)$$

$$\vdots$$

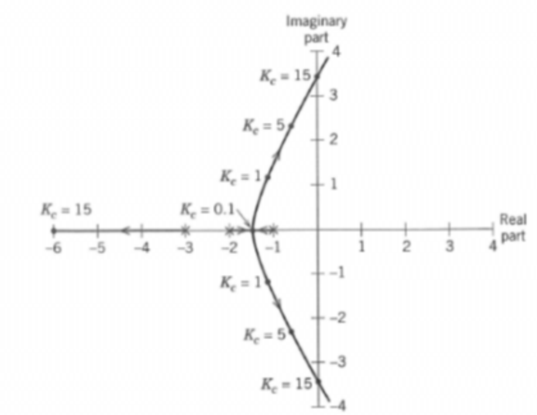
$$c_1 = \frac{b_1 a_{n-3} - a_{n-1} b_2}{b_1} \qquad (11\text{-}96)$$

$$c_2 = \frac{b_1 a_{n-5} - a_{n-1} b_3}{b_1} \qquad (11\text{-}97)$$

- Direct Substitution
  - Substitute s=jw
  - Find wc and Kcu that make real and imaginary parts equal to zero
  - May need Euler's Identity for Time Delays

$$e^{-j\omega\theta} = \cos(\omega\theta) - j\sin(\omega\theta)$$

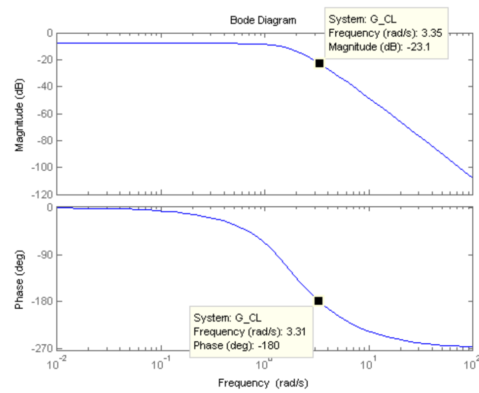
- Root Locus - Closed Looop Stable for Poles in Left-Hand Plane



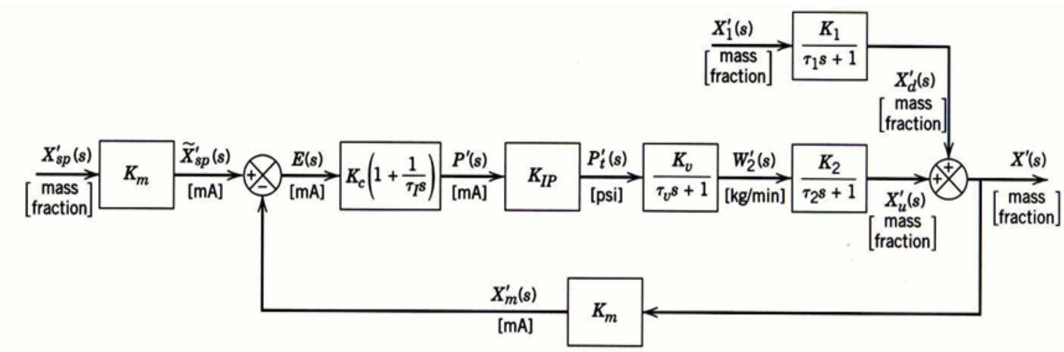
- Bode Plot Analysis

- Critical frequency ω_c is ω for which $\phi_{OL}(\omega)$=-180°
- Amplitude Ratio at Critical Frequency
  - Stable when $AR_{OL}(\omega_c) < 1$
- Decibels to Amplitude Ratio
  - Decibels = $G_{dB} = 20 \log_{10}(AR)$
  - Stability: AR < 1 or $G_{dB} < 0$
- Gain Margin

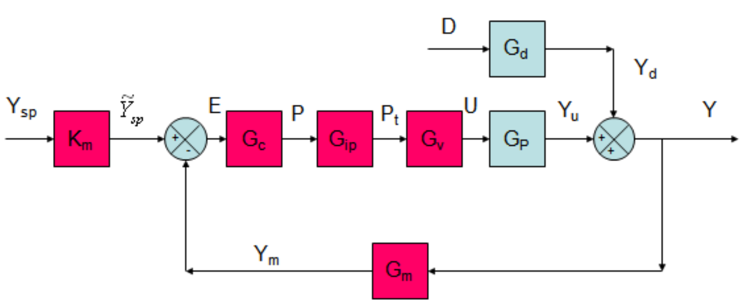
$$K_{cu} = \frac{1}{AR_G(\omega_c)} = \frac{1}{10^{\frac{G_{dB}}{20}}}$$



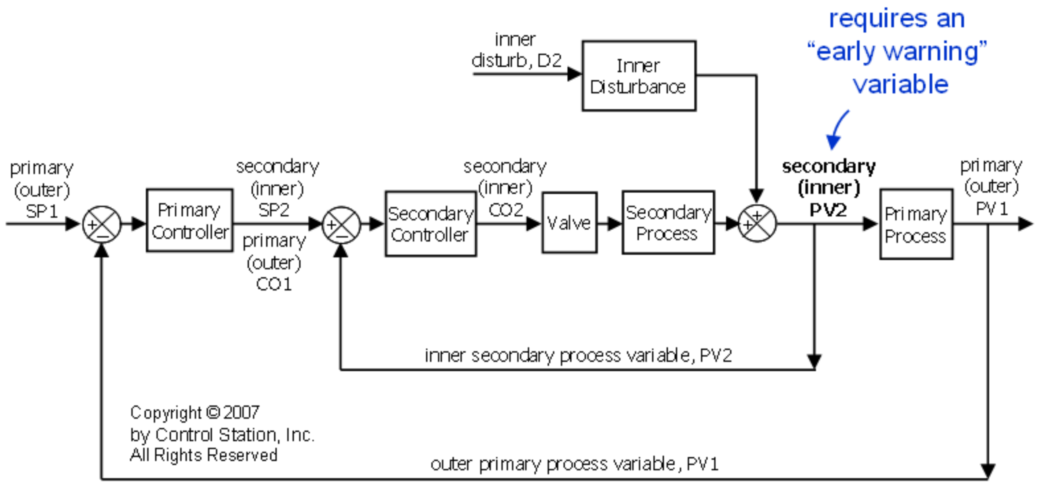
<u>Get transfer function for each piece of equipment</u>



<u>Standard Block Diagram Form</u>



<u>Cascade Control</u>



Feedforward Control

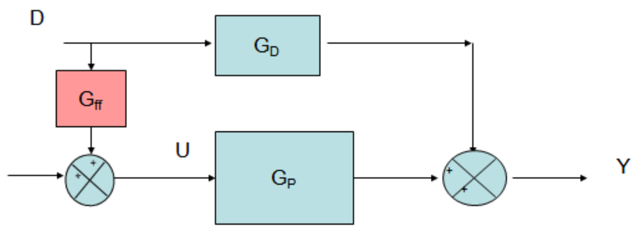
1. Write an algebraic equation for the block diagram

   $Y(s) = D(s)\cdot G_d(s) + U(s)\cdot G_p(s)$

2. If Y(s) is to be unaffected by D(s), then we want Y(s) = 0
3. Solve for U(s) in terms of D(s)

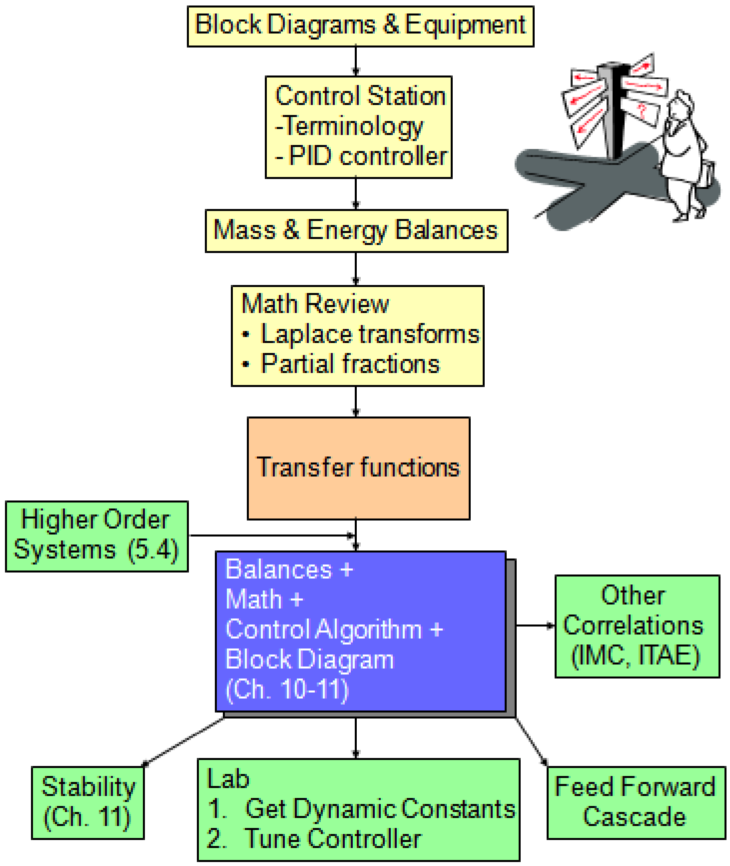
   $U(s) = [-G_d(s)/G_p(s)]\cdot D(s)$

   So $G_{ff} = -G_d(s)/G_p(s)$



Course Overview

Big Picture



Block Diagrams & Equipment
Control Station
-Terminology
- PID controller
Mass & Energy Balances
Math Review
• Laplace transforms
• Partial fractions
Transfer functions
Higher Order Systems (5.4)
Balances +
Math +
Control Algorithm +
Block Diagram
(Ch. 10-11)
Other Correlations (IMC, ITAE)
Stability (Ch. 11)
Lab
1. Get Dynamic Constants
2. Tune Controller
Feed Forward
Cascade