

ChE 436 – Process Control  
Stability Analysis Worksheet

**Example 11.13 (modified)**

Consider a feedback control system that has the open-loop transfer function,

$$G_{OL}(s) = \frac{4K_c}{(s+1)(s+2)(s+3)} \quad (11-108)$$

Determine the values of  $K_c$  that keep the closed loop system response stable.

Part a) Derive the characteristic equation (denominator of the closed loop response).

Part b) Using Routh Array analysis, determine the critical gain ( $K_{cu}$ ).

$$a_n s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_0 = 0 \quad (11-93)$$

$$a_n > 0$$

$$b_1 = \frac{a_{n-1}a_{n-2} - a_n a_{n-3}}{a_{n-1}} \quad (11-94)$$

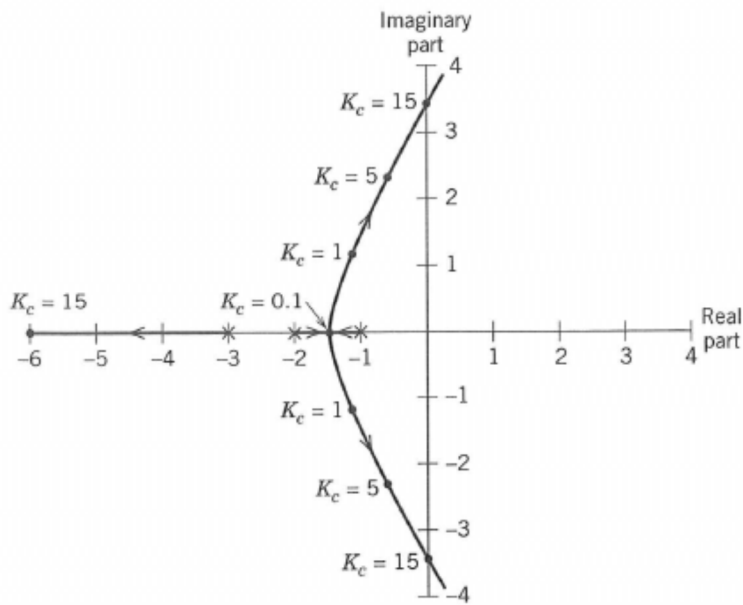
$$b_2 = \frac{a_{n-1}a_{n-4} - a_n a_{n-5}}{a_{n-1}} \quad (11-95)$$

Row	$a_n$	$a_{n-2}$	$a_{n-4}$	$\dots$
1	$a_n$	$a_{n-2}$	$a_{n-4}$	$\dots$
2	$a_{n-1}$	$a_{n-3}$	$a_{n-5}$	$\dots$
3	$b_1$	$b_2$	$b_3$	$\dots$
4	$c_1$	$c_2$	$\dots$	$\dots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$n + 1$	$z_1$	$\dots$	$\dots$	$\dots$

$$c_1 = \frac{b_1 a_{n-3} - a_{n-1} b_2}{b_1} \quad (11-96)$$

$$c_2 = \frac{b_1 a_{n-5} - a_{n-1} b_3}{b_1} \quad (11-97)$$

Part c) Using the Root Locus Plot, determine the critical gain ( $K_{cu}$ ).



Part d) Using the Bode Plot, determine the critical gain ( $K_{cu}$ ).

