Class 28

Block Diagram Algebra

Review

- Prob 11.11
  - Valve model: Step response y(t) = KM (1-exp(-t/tau))
    - t = 1 tau = 0.63
    - t = 2 tau = 0.86
    - t = 3 tau = 0.95
    - t = 4 tau = 0.98
    - t = 5 tau = 0.9933
- All variables on the block diagram are in
  - Deviation variables
  - Laplace coordinates
- PDC textbook <u>always</u> puts the comparator in mA
  - Not the case with digital systems

Offset

- Offset = Y_{sp} – Y as t $\rightarrow \infty$
- Using final value theorem:

$$Offset = \lim_{s\to 0}\left[s\cdot\left(Y_{sp} - Y\right)\right]$$

- For step change M, Laplace if (M/s)
- Assume first order process

$$Y(s) = \frac{K}{\tau s + 1}$$

$$Offset = \lim_{s\to 0}\left[s\cdot\left(\frac{M}{s} - \frac{M}{s}\frac{K}{\tau s+1}\right)\right] = M\cdot(1 - K)$$

- If not first order

$$Offset = M - \lim_{s\to 0}\left[\frac{Y}{Y_{sp}}\right]$$

Example Problem #11.5

11.5: A control system has the following transfer functions:

$G_c = 1$

$G_v = 2$

$G_p = 2/(s(s+1))$

$G_d = 2/(s(s+1))$

$G_m = 1$

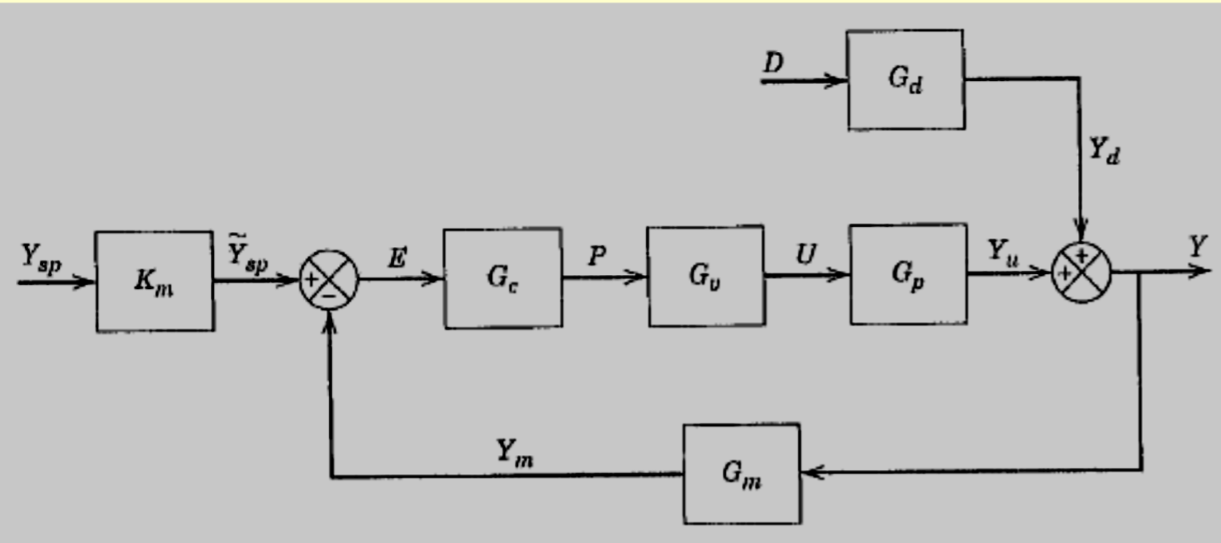


Figure 11.8 Standard block diagram of a feedback control system.

Solution a)

(a)

From Eq. 11-26 we get the closed loop transfer function for set point changes

$$\frac{Y}{Y_{sp}} = \frac{K_m G_c G_v G_p}{1 + G_c G_v G_p G_m}$$

Substituting the information from the problem gives

$$\frac{Y}{Y_{sp}} = \frac{\dfrac{4}{s(s+1)}}{1 + \dfrac{4}{s(s+1)}} = \frac{4}{s(s+1)+4} = \frac{4}{s^2+s+4}$$

Or in standard form (Eq. 5-40), with $\tau = \frac{1}{2}$ and $\zeta = \frac{1}{4}$

$$\frac{Y}{Y_{sp}} = \frac{1}{\frac{1}{4}s^2 + \frac{1}{4}s + 1}$$

Solution b)

(b)

Given a unit step change in set point we obtain

$$Y(s) = \frac{4}{s(s^2 + s + 4)}$$

Using the Final Value Theorem we get

$$\lim_{s \longrightarrow 0} sY(s) = \frac{4}{s^2 + s + 4} = \frac{4}{4} = 1$$

Therefore $y(\infty) = 1$

Solutions c and d)

(c)

As the step change is a unit step change, and we have shown that $y(\infty) = 1$, we can say that offset = 0. This is consistent with the fact that the gain of the overall transfer function is 1, so no offset will occur. Normally proportional control does not eliminate offset, but it does for this integrating process.

(d)

Using Eq. 5-51 or taking the inverse Laplace transform of the response given

above we get

$$y(t) = 1 - e^{-t/2}\left[\cos\left(\frac{\sqrt{15}}{2}t\right) + \frac{\sqrt{15}}{15}\sin\left(\frac{\sqrt{15}}{2}t\right)\right]$$

Substituting the value of 0.5 for t gives

$$y(0.5) = 0.39295$$

Solution e)

(e)

We can tell from the response derived above that the response will be oscillatory. The fact that the roots of the characteristic equation are imaginary confirms this, as does the fact that $0 < \zeta < 1$

Solution f)

$$G_{CL} = \frac{Y(s)}{Y_{sp}(s)} = \frac{4/5}{1/5s+1}$$

$$offset = \lim_{s->0}(s(Y_{sp}(s) - Y(s)))$$

$$offset = \lim_{s->0}\left(sY_{sp}(s)\left(1 - \frac{4/5}{1/5s+1}\right)\right)$$

$$substitute\, Y_{sp}(s) = \frac{M}{s}$$

$$offset = \lim_{s->0}\left(M\left(1 - \frac{4/5}{1/5s+1}\right)\right)$$

$$offset = M\left(1 - 4/5\right) = M\,/\,5$$

Example Problem #2

11.5: A control system has the following transfer functions:

$G_c = 1$

$G_v = 2$

$G_p = 2/(s+1)$

$G_m = 1$

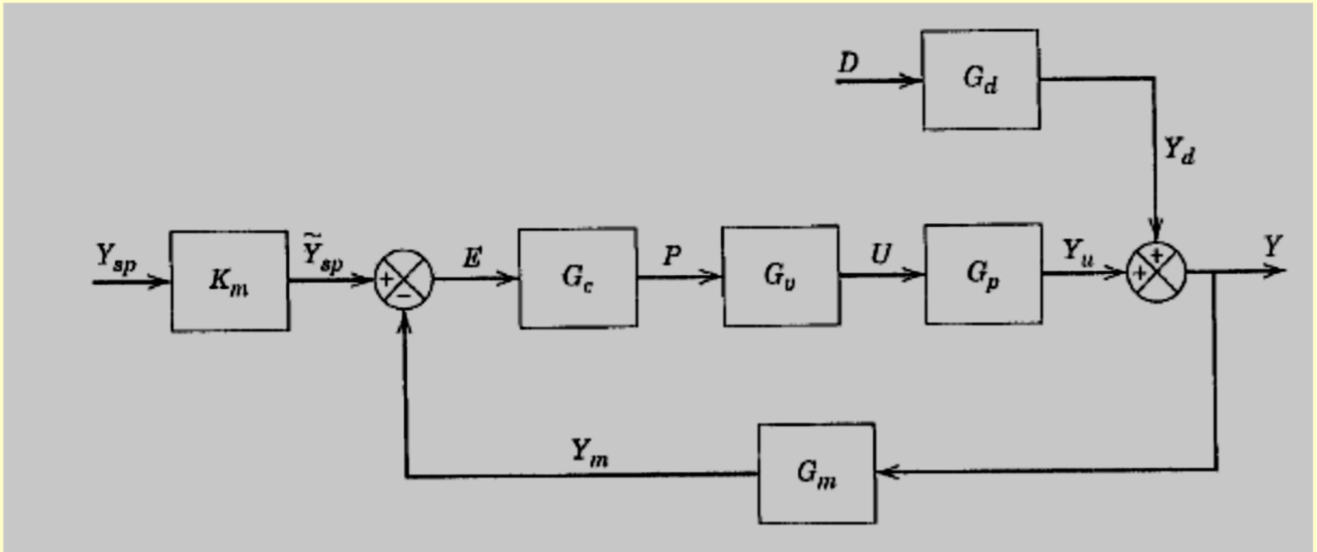
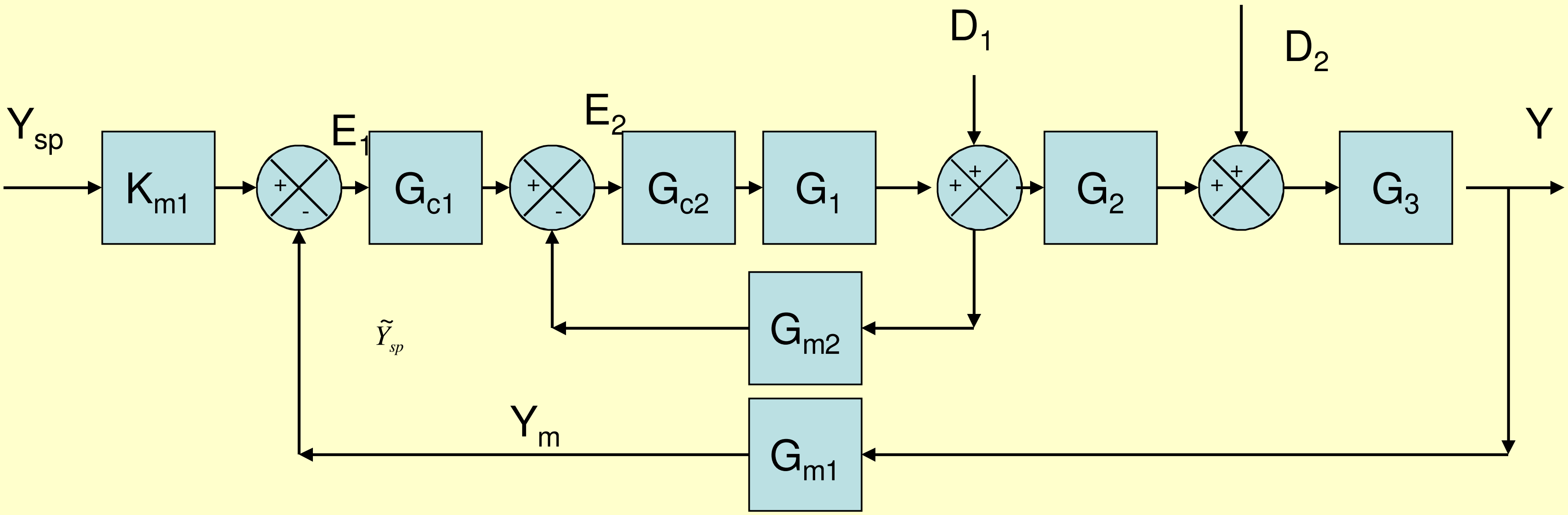


Figure 11.8 Standard block diagram of a feedback control system.

Compute the steady-state offset (y_{sp}-y)

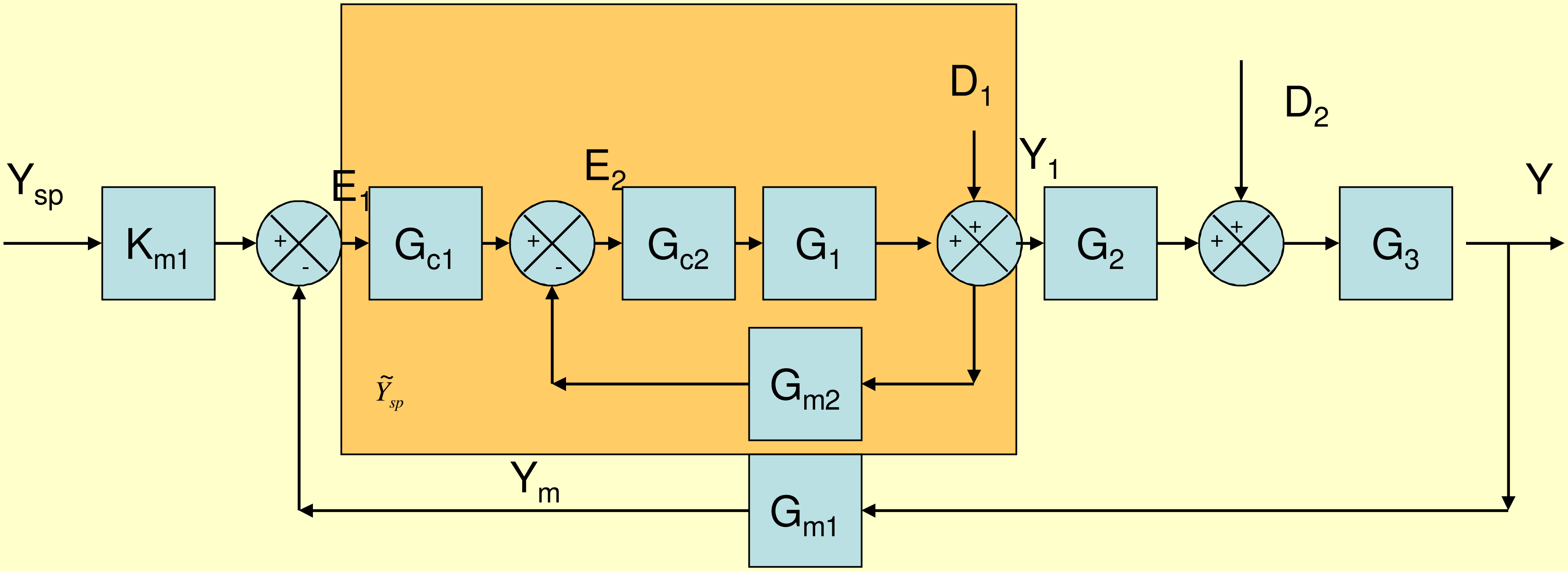
Example Problem #3



Wanted: $\frac{Y}{Y_{sp}}$

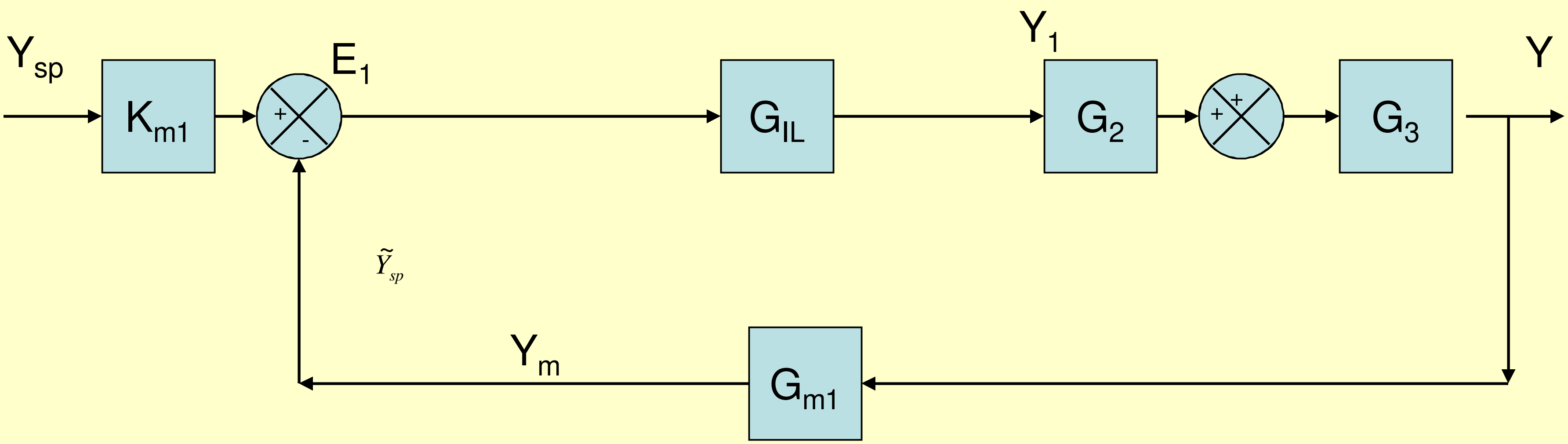
Yikes!!!

Inner Loop



$$\frac{Y_1}{E} = G_{IL} = \frac{direct}{1 + loop} = \frac{G_{c1}G_{c2}G_1}{1 + G_{m2}G_{c2}G_1}$$

Outer Loop



$$\frac{Y}{Y_{sp}} = \frac{K_{m1} G_{IL} G_2 G_3}{1 + G_{m1} G_{IL} G_2 G_3}$$

Substitute & Rearrange

$$\frac{Y}{Y_{sp}} = \frac{K_{m1}G_{IL}G_2G_3}{1 + G_{m1}G_{IL}G_2G_3}$$

$$\frac{Y}{Y_{sp}} = \frac{K_{m1}\dfrac{G_{c1}G_{c2}G_1}{1 + G_{m2}G_{c2}G_1}G_2G_3}{1 + G_{m1}\dfrac{G_{c1}G_{c2}G_1}{1 + G_{m2}G_{c2}G_1}G_2G_3}$$

$$\frac{Y}{Y_{sp}} = \frac{K_{m1}G_{c1}G_{c2}G_1G_2G_3}{1 + G_{m2}G_{c2}G_1 + G_{m1}G_{c1}G_{c2}G_1G_2G_3}$$