Zero-Pole Plots in Mathcad

Mathcad symbolic functions can be very useful to obtain information about transfer functions, the inherent stability of processes, and responses to various input functions.

Consider the transfer function: $G(s) := \dfrac{5\cdot(2\cdot s - 1)}{(3\cdot s + 1)\cdot(4\cdot s + 1)\cdot\left(s^2 + 4\right)}$

It is convenient to divide the transfer function up into a separate numerator and denominator in order to look at the poles and zeros.

$$N(s) := 5\cdot(2\cdot s - 1) \qquad D(s) := (3\cdot s + 1)\cdot(4\cdot s + 1)\cdot\left(s^2 + 4\right)$$

To obtain the roots, take the denominator and find the polynomial coefficients using the symbolic keyword coeffs (press control + shift + period and type coeffs).

$$v := D(s)\ \text{coeffs} \ \rightarrow \begin{pmatrix} 4 \\ 28 \\ 49 \\ 7 \\ 12 \end{pmatrix}$$

Another keyword that you might occasionally find useful is the expand function

$$D(s)\ \text{expand} \ \rightarrow 12\cdot s^4 + 7\cdot s^3 + 49\cdot s^2 + 28\cdot s + 4$$

This keyword will show you the full polynomial form of the function. Note that the vector created with the coeffs keyword lists the coeffeicients in the order of the power on s from lowest to highest; i.e., it starts with the constant and ends with the highest order of the polynomial.

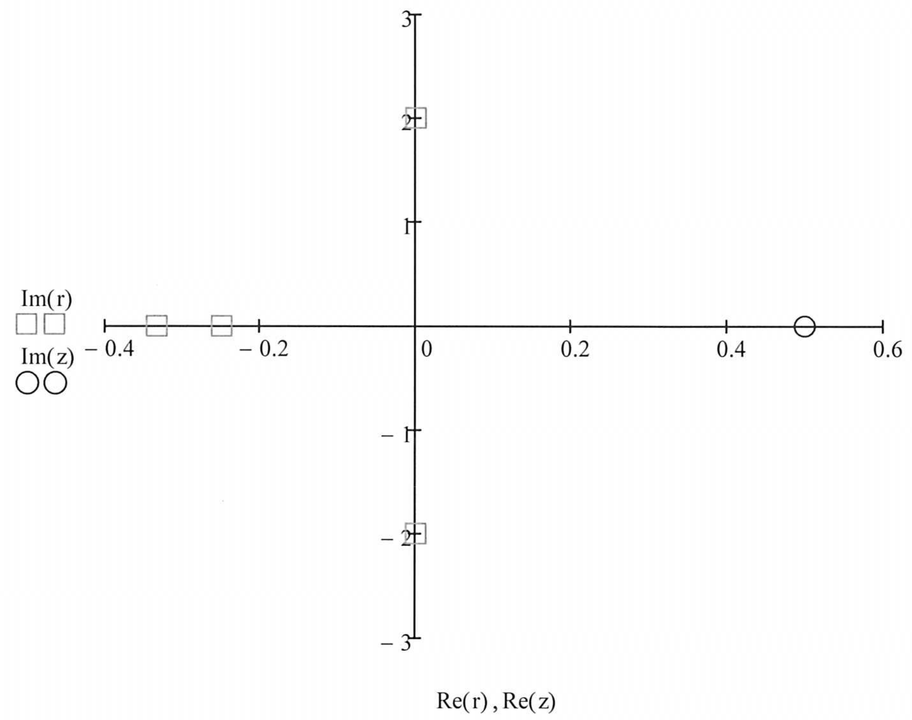
Now we will find all of the poles of the transfer function using the polyroots function on the denominator.

$$r := \text{polyroots}(v) \qquad r = \begin{pmatrix} -0.333 \\ -0.25 \\ -1.022 \times 10^{-9} - 2i \\ -1.022 \times 10^{-9} + 2i \end{pmatrix}$$

Zeros are obtained by performing the same procedures on the numerator.

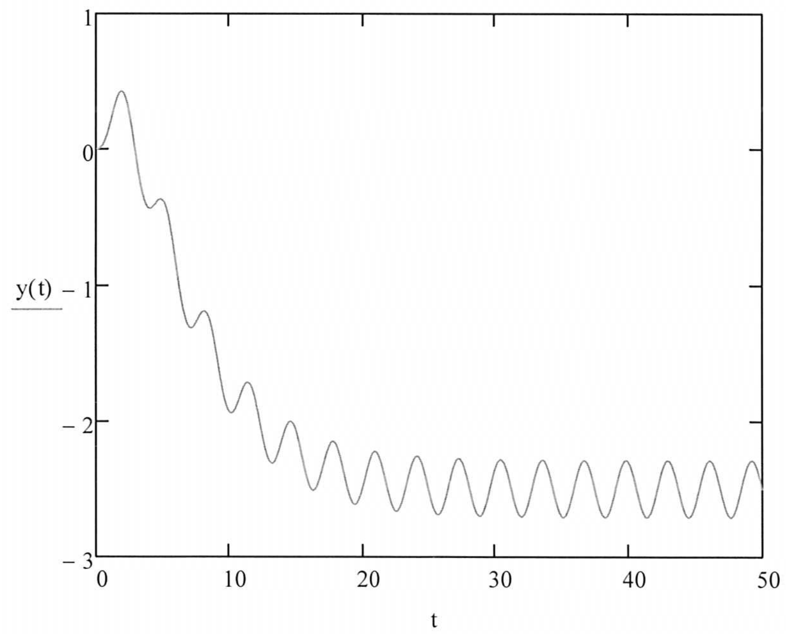
$$v := N(s)\ \text{coeffs} \ \rightarrow \begin{pmatrix} -5 \\ 10 \end{pmatrix} \qquad z := \text{polyroots}(v)$$

To create a plot of the zeros and poles in the complex plane, we will use the Im and Re functions, which return the imaginary and real portions of a complex number, respectively.



This zero-pole plot indicates a purely oscillatory component to the solution. Let's plot the response to a step function input of magnitude 2.

$$y(t) := G(s)\cdot\frac{2}{s} \text{ invlaplace}, s, t \rightarrow \frac{192\cdot e^{-\frac{t}{4}}}{13} - \frac{87\cdot\sin(2\cdot t)}{481} - \frac{450\cdot e^{-\frac{t}{3}}}{37} - \frac{103\cdot\cos(2\cdot t)}{962} - \frac{5}{2}$$



Here are some additional symbolic keywords and techniques that you might find useful in analyzing transfer functions in Mathcad.

A. Partial fractions

$$\frac{1}{x^2 + 6\cdot x + 5} \text{ parfrac } \rightarrow \frac{1}{4\cdot(x+1)} - \frac{1}{4\cdot(x+5)}$$

B. Simplify

$$e^{-2\cdot s} - e^{2\cdot s} \text{ simplify } \rightarrow -2\cdot\sinh(2\cdot s)$$

C. Collect (collects terms of like powers)

$$b(s) := 5\cdot s^2 + 3\cdot s + 2 + 6\cdot s^3 + 4\cdot s^2 + s + 1$$

$$b(s) \text{ collect } \rightarrow 6\cdot s^3 + 9\cdot s^2 + 4\cdot s + 3$$

D. Float (truncates numerical results to the indicated number of digits)

$$8.433982569234528\cdot s \text{ float},4 \rightarrow 8.434\cdot s$$

E. Factor

$$x^2 + x + 2 \text{ factor, domain} = \text{complex} \rightarrow (x + 0.5 - 1.3228756555322952953i)\cdot(x + 0.5 + 1.322875655532295$$

Note: Mathcad will try to factor things in the real domain by default, so the domain=complex term should be included if you expect complex factors.

$$x^2 + x + 2 \text{ factor } \rightarrow x^2 + x + 2$$ no factors in the real domain

F. Multiple keywords. Multiple keywords can be used sequentially by typing CNTRL+SHIFT+Period a second time (once to set oup the keyword place holder and the second time to create the vertical line with an additional place holder).

$$x^2 + x + 2 \left| \begin{array}{l} \text{factor, domain} = \text{complex} \\ \text{float},3 \end{array} \right. \rightarrow (x + 0.5 - 1.32i)\cdot(x + 0.5 + 1.32i)$$

$$G(s) \left| \begin{array}{l} \text{invlaplace} \\ \text{simplify} \end{array} \right. \rightarrow \frac{103\cdot\sin(2\cdot t)}{962} - \frac{87\cdot\cos(2\cdot t)}{481} + \frac{75\cdot e^{-\frac{t}{3}}}{37} - \frac{24\cdot e^{-\frac{t}{4}}}{13}$$

G. Solve (Used to find roots of equations and solve single-variable expressions)

$$x^2 + x + 2 \text{ solve } \rightarrow \begin{bmatrix} -\frac{1}{2} - \left(\frac{\sqrt{7}}{2}\right)\cdot i \\ -\frac{1}{2} + \frac{1}{2}\cdot\sqrt{7}\cdot i \end{bmatrix}$$