Name: ________________________

(Adapted from a Group Exercise developed by D. Clough at the University of Colorado at Boulder)

Poles and Zeros

A transfer function can usually be represented as a ratio of two polynomials in the Laplace variable s along with a possible delay term. This can be written as

$$G(s) = \frac{Z(s)}{P(s)} e^{-\theta s}$$

where $Z(s) = b_m s^m + b_{m-1} s^{m-1} + \ldots + b_1 s + b_0$

and $P(s) = a_n s^n + a_{n-1} s^{n-1} + \ldots + a_1 s + a_0$

The roots of $Z(s)$ are called the zeros of the transfer function (when "s" takes on one of those root values, the transfer function goes to zero). The roots of $P(s)$ are called the poles of the transfer function (as s approaches one of those roots, the transfer function grows without bound -- this singularity looks like a pole, well sort of). Note that in order to be "physically realizable," n $\geq$ m (this will be important later).

Each of the polynomials can be factored to give the following form

$$G(s) = \frac{b_m (s - z_1)(s - z_2)\ldots(s - z_m)}{a_n (s - p_1)(s - p_2)\ldots(s - p_n)} e^{-\theta s}$$

where $z_1, z_2, \ldots, z_m$ are the zeros

and $p_1, p_2, \ldots, p_n$ are the poles.

Note: These values may be real or complex. If they are complex, they will occur as conjugate pairs.

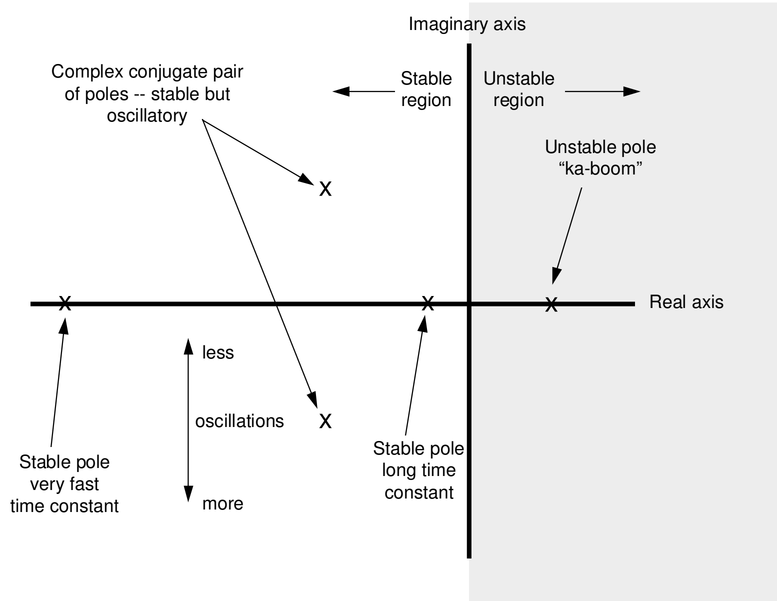
If all the zero and pole values are factored out, the standard "time constant" form results:

$$G(s) = \frac{b_m(-z_1)(-z_2)\ldots(-z_m)(\tau_{l1}s+1)(\tau_{l2}s+1)\ldots(\tau_{lm}s+1)}{a_n(-p_1)(-p_2)\ldots(-p_n)(\tau_1 s+1)(\tau_2 s+1)\ldots(\tau_n s+1)} e^{-\theta s}$$

$$= K \frac{(\tau_{l1}s+1)(\tau_{l2}s+1)\ldots(\tau_{lm}s+1)}{(\tau_1 s+1)(\tau_2 s+1)\ldots(\tau_n s+1)} e^{-\theta s}$$

The pole and zero values govern the dynamic behavior of a process. In particular, the pole values determine the stability of the process.

Pole and zero values are usually depicted on the complex plane (since there may be complex conjugate pairs). Poles are shown by $\times$'s and zeros by O's. The diagram on the reverse helps in the interpretation of dynamics according to pole location.



Consider the transfer function:

$$G(s) = \frac{30}{24s^3 + 20s^2 + 10s + 2} \Rightarrow \frac{15}{12s^3 + 10s^2 + 5s + 1} \Rightarrow \frac{15}{(3s+1)(4s^2 + 2s + 1)} \quad (1)$$

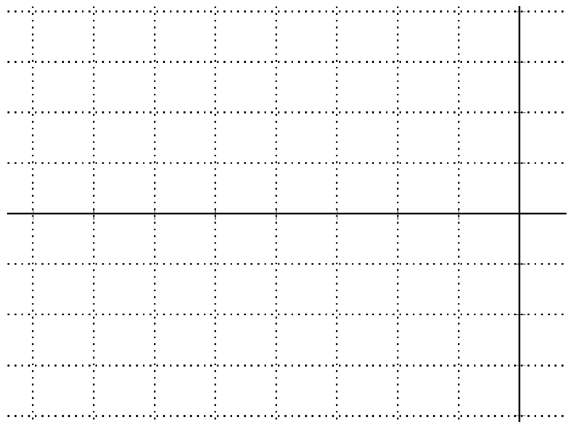
Express this transfer function in pole-zero form:

Convert the quadratic function into the form required for the denominator of the sine function in Laplace coordinates (#17 in Table 3.1, page 54). Hint: complete the square.

What are the poles (b and ω)? (two values of b, one value of ω)

Mathcad is used to find the roots of this polynomial which are: $\left(-\frac{1}{3}, \frac{-1+j\sqrt{3}}{4}, \frac{-1-j\sqrt{3}}{4}\right)$.

Plot the poles and zeros on the complex plane below (tic marks are in increments of 0.1):



Comment on the locations of the poles.