**Methods for Computing Laplace Transforms**

$$F(s) = \mathrm{L}\left[f(t)\right] = \int_0^{\infty} f(t)\, e^{-st}\, dt$$

<u>1. Derive the Laplace Transform from the Definition</u>

$$L(a) = \int_0^{\infty} a e^{-st}\, dt =$$

$$L\left(\frac{df}{dt}\right) = \int_0^{\infty} \frac{df}{dt} e^{-st}\, dt =$$

Hint: Integration by Parts (derive from the Chain Rule)

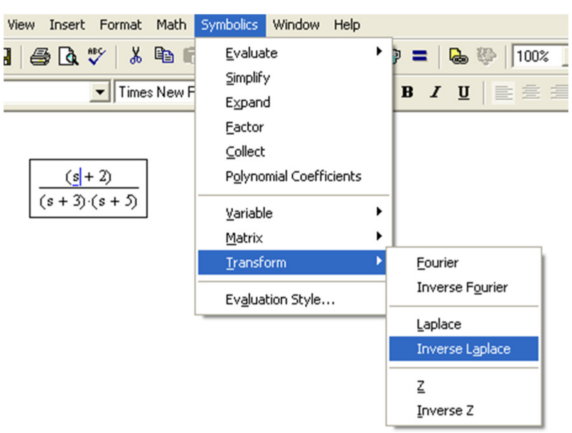
- Chain Rule: $\dfrac{\partial (uv)}{\partial t} = u\dfrac{\partial v}{\partial t} + v\dfrac{\partial u}{\partial t}$
- Rearrange: $u\dfrac{\partial v}{\partial t} = \dfrac{\partial (uv)}{\partial t} - v\dfrac{\partial u}{\partial t}$
- Integration by Parts: $\int u\dfrac{\partial v}{\partial t} = uv - \int v\dfrac{\partial u}{\partial t}$

<u>2. Use the Tables on pg. 42-43 of PDC</u>

**Table 3.1** Laplace Transforms for Various Time-Domain Functions[a]

| $f(t)$ | | $F(s)$ |
|---|---|---|
| 1. $\delta(t)$ | (unit impulse) | $1$ |
| 2. $S(t)$ | (unit step) | $\dfrac{1}{s}$ |
| 3. $t$ | (ramp) | $\dfrac{1}{s^2}$ |
| 4. $t^{n-1}$ | | $\dfrac{(n-1)!}{s^n}$ |
| 5. $e^{-bt}$ | | $\dfrac{1}{s+b}$ |
| 6. $\dfrac{1}{\tau} e^{-t/\tau}$ | | $\dfrac{1}{\tau s + 1}$ |
| 7. $\dfrac{t^{n-1}e^{-bt}}{(n-1)!}$ | $(n > 0)$ | $\dfrac{1}{(s+b)^n}$ |
| 8. $\dfrac{1}{\tau^n (n-1)!} t^{n-1} e^{-t/\tau}$ | | $\dfrac{1}{(\tau s + 1)^n}$ |
| 9. $\dfrac{1}{b_1 - b_2}(e^{-b_2 t} - e^{-b_1 t})$ | | $\dfrac{1}{(s+b_1)(s+b_2)}$ |
| 10. $\dfrac{1}{\tau_1 - \tau_2}(e^{-t/\tau_1} - e^{-t/\tau_2})$ | | $\dfrac{1}{(\tau_1 s + 1)(\tau_2 s + 1)}$ |
| 11. $\dfrac{b_3 - b_1}{b_2 - b_1} e^{-b_1 t} + \dfrac{b_3 - b_2}{b_1 - b_2} e^{-b_2 t}$ | | $\dfrac{s + b_3}{(s+b_1)(s+b_2)}$ |
| 12. $\dfrac{1}{\tau_1}\dfrac{\tau_1 - \tau_3}{\tau_1 - \tau_2} e^{-t/\tau_1} + \dfrac{1}{\tau_2}\dfrac{\tau_2 - \tau_3}{\tau_2 - \tau_1} e^{-t/\tau_2}$ | | $\dfrac{\tau_3 s + 1}{(\tau_1 s + 1)(\tau_2 s + 1)}$ |
| 13. $1 - e^{-t/\tau}$ | | $\dfrac{1}{s(\tau s + 1)}$ |

<u>3. Use MATHCAD</u>



Practice Problems

a) 1000 **S**(t)  (Step function with a magnitude of 1000)

b) $5e^{-6t} + \sin 4t + 5$

c) $\dfrac{d^3y}{dt^3} \quad where \left(\dfrac{d^2y}{dt^2}\right)_{t=0} = 0, \left(\dfrac{dy}{dt}\right)_{t=0} = 2, \quad y(0) = 3$

d) $\dfrac{dy}{dt} + 3y = e^{-2t} \quad y(0) = 2$

1. Take the L of both sides of the ODE.
2. Rearrange the resulting algebraic equation in the *s* domain to solve for the L of the output variable, e.g., *Y*(*s*).
3. Perform a partial fraction expansion.
4. Use the $L^{-1}$ to find *y*(*t*) from the expression for *Y*(*s*).
5. Check your answer by substituting y(t) and y'(t) into the original equation.