Worksheet on Linearization

Example 1. Linearization with one variable

Linearize the following equation around $\bar{x}$ =3:

$$f(x) = 3x^3 + 5x^2 + 27$$

(i) Write the Taylor's series expansion:

f(x) =

(ii) Evaluate $f(\bar{x})$ =

(iii) What is the derivative of the function? $\left(\frac{df}{dx}\right)$=

(iv) Evaluate $f'(\bar{x})$ =

(v) Write the final linear expression *f(x)* =

Example 2. Linearization with two variables

Linearize the following equation around $\bar{x} = 2$ and $\bar{y} = 2$:

$$f(x, y) = 3xy + y^2 - 3x^2$$

(i) Write the Taylor's series expansion:

f(x,y) =

(ii) Evaluate $f(\bar{x}, \bar{y})$ =

(iii) What are the partial derivatives of the function?

$\frac{\partial f}{\partial x} =$

$\frac{\partial f}{\partial y} =$

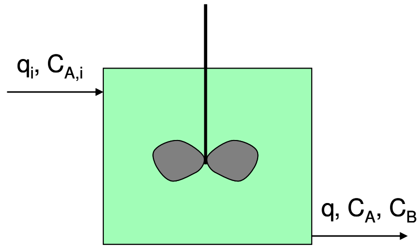
(iv) Evaluate $\left.\frac{\partial f}{\partial x}\right|_{\bar{x},\bar{y}} =$

$\left.\frac{\partial f}{\partial y}\right|_{\bar{x},\bar{y}} =$

(v) Write the final linear expression:

f(x) =

Example 3. CSTR with three variables



$-r_A = k_1 C_A^2 - k_2 C_A C_B$

(i) Write the transient mole balance for species A:

$$V\frac{dC_A}{dt} =$$

(ii) Assume k_1, k_2, q_i, q, and V are constant. The function to linearize is just the RHS of the above equation! The variables are ___, ____, and ____.

(iii) At steady state, what is the value of $V\frac{dC_A}{dt}$? ____.

Therefore, $f\left(\overline{C}_{A,i}, \overline{C}_A, \overline{C}_B\right) =$ ____.

(iv) Write the Taylor's series expansion:

$$f\left(C_{A,i}, C_A, C_B\right) =$$

(v) What are the partial derivatives of the function?

$$\frac{\partial f}{\partial C_{A,i}} =$$

$$\frac{\partial f}{\partial C_A} =$$

$$\frac{\partial f}{\partial C_B} =$$

(vi) Evaluate $\left.\frac{\partial f}{\partial C_{A,i}}\right|_{ss} =$

$$\left.\frac{\partial f}{\partial C_A}\right|_{ss} =$$

$$\left.\frac{\partial f}{\partial C_B}\right|_{ss} =$$

(vii) Write the final linear expression:

$$f\left(C_{A,i}, C_A, C_B\right) = \_\_\_ + \_\_\_\left(C_{A,i} - \overline{C}_{A,i}\right) + \_\_\_\_\_\_\_\_\_\_\left(C_A - \overline{C}_A\right) + \_\_\_\_\_\_\left(C_B - \overline{C}_B\right)$$

(viii) Now introduce deviation variables (the prime here is not a derivative):

$$C'_{A,i} = C_{A,i} - \overline{C}_{A,i}$$

$$C'_A = C_A - \overline{C}_A$$

$$C'_B = C_B - \overline{C}_B$$

(ix) The transient linearized equation now becomes:

$$V\frac{dC_A}{dt} = V\frac{dC'_A}{dt} = \underline{\quad\quad} C'_{A,i} + \underline{\quad\quad\quad\quad\quad\quad\quad\quad} C'_A + \underline{\quad\quad\quad\quad\quad} C'_B$$

(x) All of the underlined terms above are constants, since they were evaluated at the steady-state condition. For convenience in this equation, call the second constant c_1. The standard form for solving this equation using Laplace transforms is:

$$\tau\frac{dy'}{dt} + y' = f(x_1, x_2, x_3, etc.)$$

Put the equation in (ix) into standard form:

$$\frac{V}{c_1}\frac{dC'_A}{dt} + C'_A = \underline{\quad\quad\quad\quad} C'_{A,i} + \underline{\quad\quad\quad\quad} C'_B$$