Mathematical Modeling Case Study: Friction Stir Welding

Develop a Dynamic Model

- Draw a schematic diagram, labeling process variables
- List all assumptions
- Classify Problem
  - Time Dependence Only
    - ODE: Ordinary differential equations
    - DAE: Differential algebraic equations
  - Time and Spatial Dependence
    - PDE: Partial differential equations
    - PDAE: Partial differential algebraic equations
- Write dynamic balances (mass, species, energy)
- Other relations (thermo, reactions, geometry, etc.)
- Degrees of freedom
  - Does # of eqns = # of unknowns?
- Simplify

Balances

- **Total Mass Balance:**

$$\frac{dm}{dt} = \frac{d(\rho V)}{dt} = \sum_{i=inlet} \dot{m}_i - \sum_{j=outlet} \dot{m}_j$$

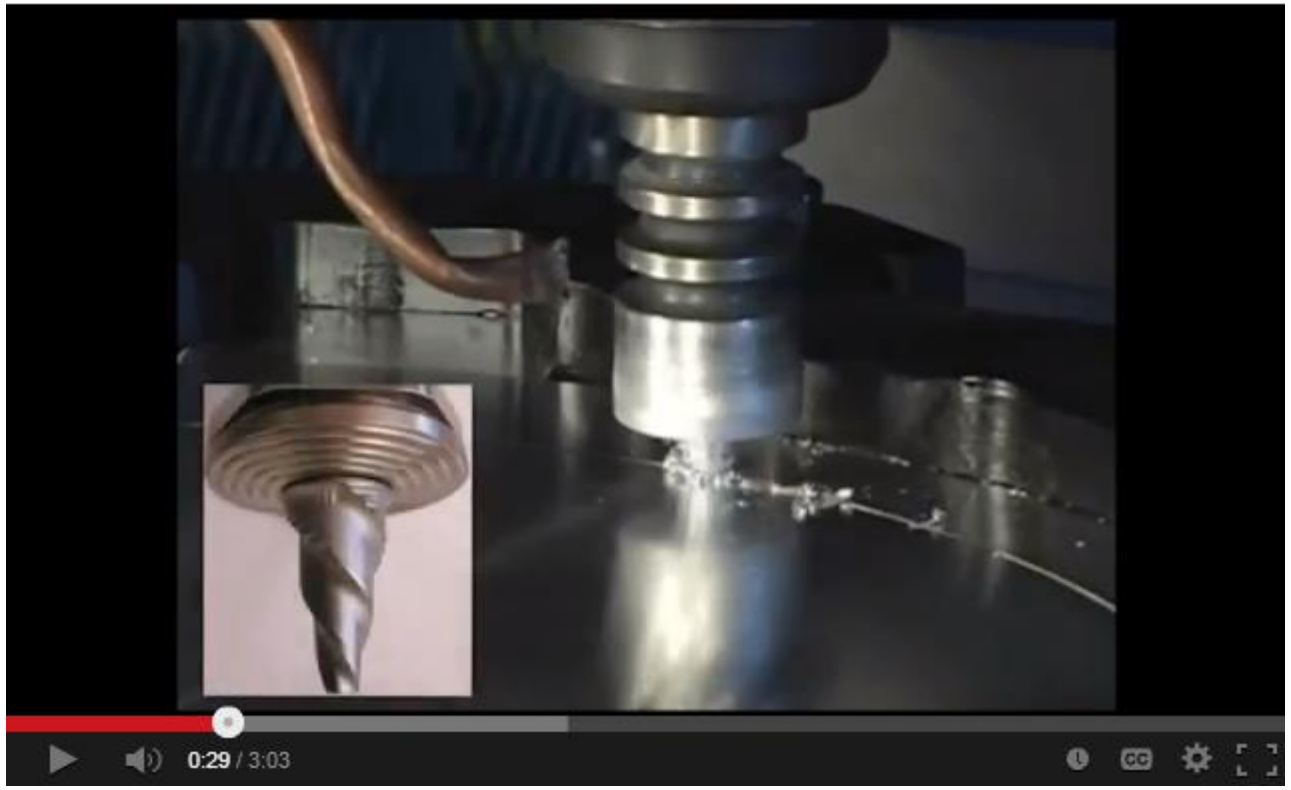
- **Species Mole Balance:**

$$\frac{dn_A}{dt} = \frac{d(c_A V)}{dt} = \sum_{i=inlet} c_{Ai} q_i - \sum_{j=outlet} c_{Aj} q_j + r_A V$$

- **Total Energy Balance:**

$$\frac{d[\rho C_p V (T - T_{ref})]}{dt} = \sum_{i:inlet} \dot{m}_i C_p (T_i - T_{ref}) - \sum_{j:outlet} \dot{m}_j C_p (T_j - T_{ref}) + Q + W_s$$

Friction Stir Welding Background



http://youtu.be/rim0wJxZ-O8

Process Diagram

Assumptions

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____
7. _____
8. _____

Additional Information

Manipulated Variables	
T_heater	Temperature at the top of the rod (K)
Disturbances	
T_amb = 298 K	Ambient temperature (K)
h = 1000	Heat transfer coefficient (W/m^2-K)
rho	Density of metal rod (kg/m^3)
Cp	Heat capacity of metal rod (J/kg-K)
k	Thermal conductivity (W/m-K)
Differential States	
T[1:20]	Temperature throughout the rod at discrete points (K)

Model Equations

Energy balance for each segment

$$\frac{dh}{dt} = \sum_{i:inlet} \dot{h}_{in} - \sum_{j:outlet} \dot{h}_{out} + Q$$

Degrees of Freedom

Number of Variables = Number of Segments

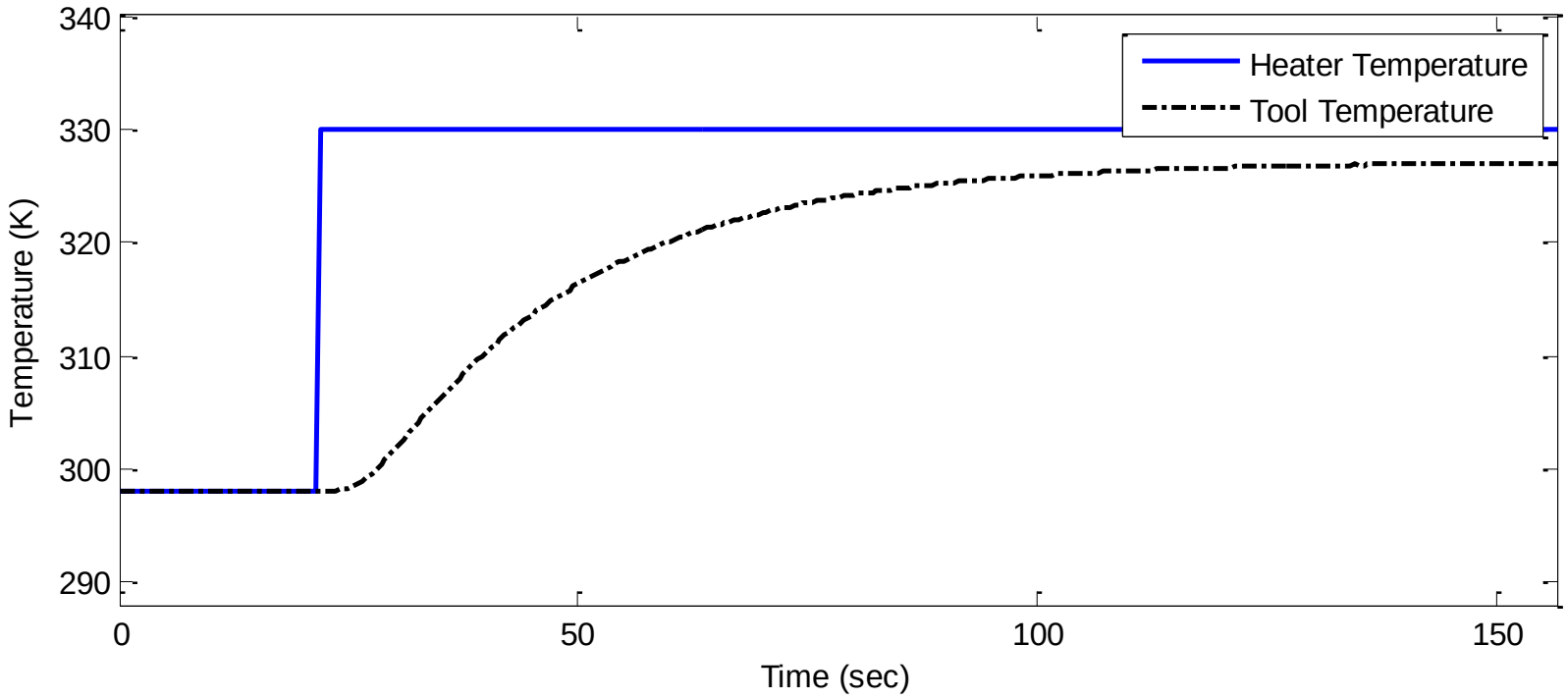
$$T_i$$

Number of Equations = Number of Segments

$$\rho C_p V \frac{dT}{dt} = \frac{k_{i-1} A}{\Delta x}(T_{i-1} - T_i) - \frac{k_i A}{\Delta x}(T_i - T_{i+1}) - hA(T_i - T_{amb})$$

$$N_{DOF} = N_{Variables} - N_{Equations}$$

Simulate: Step Test



Build Model in Simulink