# Mathcad Lecture #5 In-class Worksheet
## Plotting and Calculus - solutions

At the end of this lecture, you should be able to:
- graph expressions, functions, and matrices of data
- format graphs with titles, legends, log scales, lines, and axis labels
- find the roots of an expression by tracing
- perform differential and integral calculus numerically
- perform differential and integral calculus symbolically
- solve engineering problems involving calculus

# 1. Plotting Basics

## A. Syntax

2D plots can be inserted using by:
- typing the @ symbol
- using the graph icon on the Math/Graph tool palette

The black boxes represent placeholders.
- Must fill in the center boxes on each axis; these are the independent variable (x-axis) and the function (y-axis)
- Can fill in boxes at extrema of each axis; these are the bounds

## B. Demonstration of plotting expression.
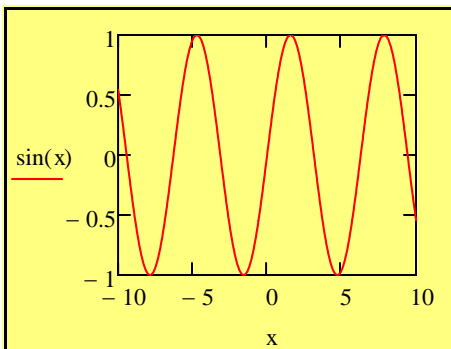


Changing the Bounds
1. clicking on the graph causes the bound to appear at the extrema of each axis
2. the bound are surrounded by small half brackets
3. *the first time you begin typing on a clicked bound, the previous contents will disappear*
4. *if you want to change the bound again, you will have to first delete the previous bound and then type the new bound.*
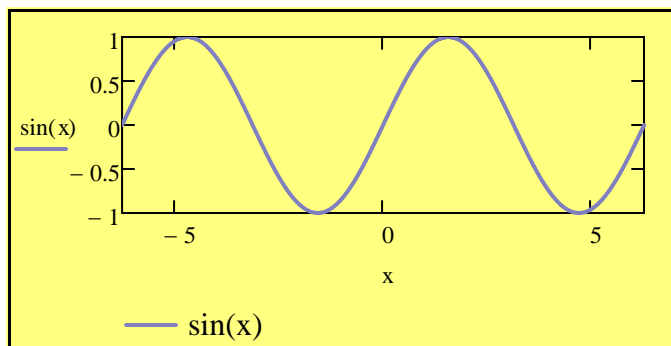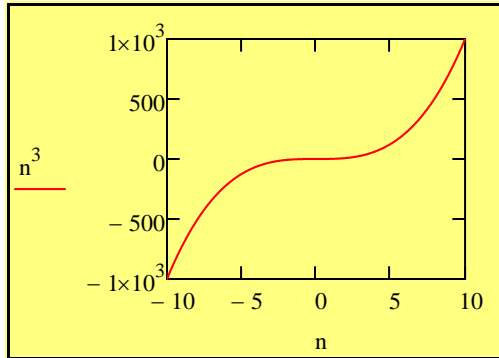


You can change a variety of plot properties by double clicking on the graph.
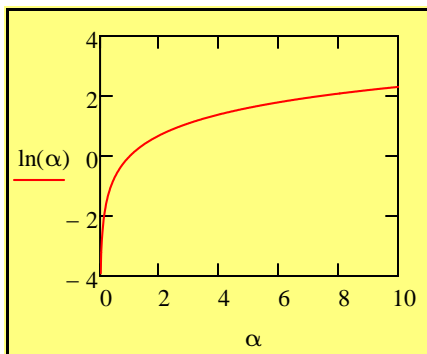
Note: demonstrate how to add a caption and change the color and weight of the line.
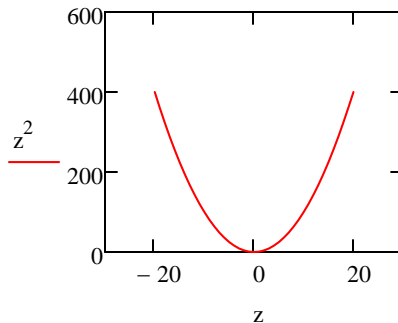
## C. Practice

Plot $n^3$ vs $n$
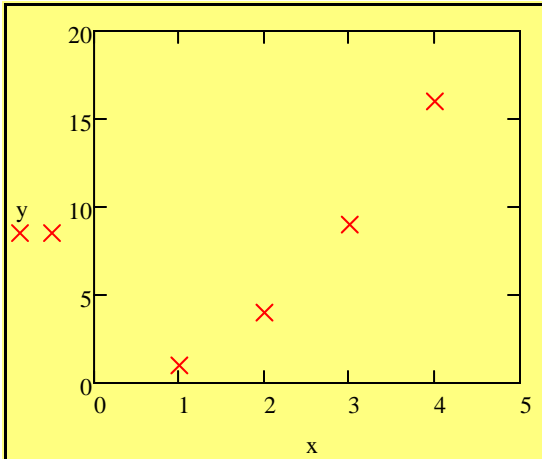


Plot $\ln(\alpha)$ vs $\alpha$.

$z := -20, -19 \, .. \, 20$

# D. Demonstration of plotting vector of data.

Explanation: We often want to graph discrete points of measured data.  This is done by placing the da
in a vector.  The vectors must be the same size.

$$i := 0 .. 3$$

$$x := \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix} \qquad y_i := \left(x_i\right)^2 \qquad y = \begin{pmatrix} 1 \\ 4 \\ 9 \\ 16 \end{pmatrix}$$
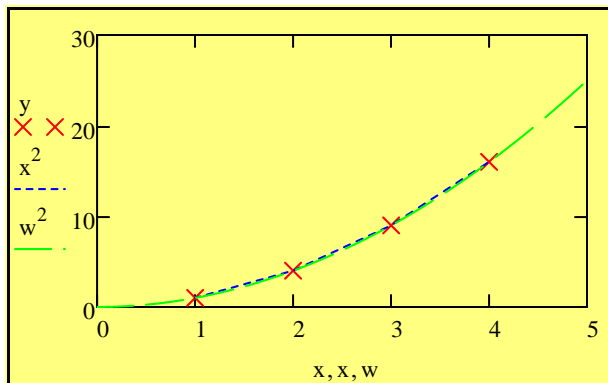


Key Points:
1.  In general, when you are plotting discrete points such as those obtained from direct experimental data, you do not use a line.
2.  A line is usually used if the graph is made from a function, a correlation, or a fit to the data.
3.  A line may be used to connect points, but it should be noted that is is only there to guide the eye.  In such cases, you should use straight lines to connect the points.

# D. Demonstration of multiple plots on the same graph.

Explanation: Sometimes, we want to include more than one set of data on the same graph.  This is done by separating the arguments with a comma.  (Typing the comma key give you a new place holder to the right of or below the previous argument.)



Key Points:
1.  Notice that each x argument is matched to a y argument.
2.  You can plot multiple expressions using the same independent variable if desired; however, type of each argument must match.
3.  You cannot match an expression (on the y axix) with a matrix of data (on the x axis) when plotting. (see below.)
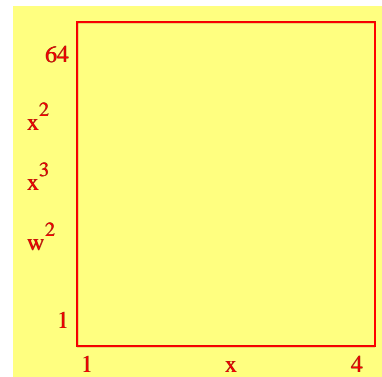
## E. Demonstration of plotting user-defined functions.

You can also plot user defined functions.

$f(b) := b^2$



## F. Practice

The vapor pressure of acetone is described well by the following expression,

$$P^{sat}(T) = \exp\left[A + \frac{B}{T} + C\ln(T) + DT^2\right]$$

where A = 69.006, B = -5599.6, C = -7.0985, and D = 6.2237E-6.  The temperature must be evaluated in Kelvin and the pressure returned is in Pascal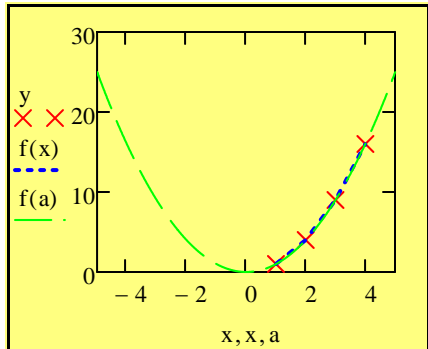.  Experimental data for the vapor pressure of acetone is available in variable found to the right.  On a graph, plot both the experimental vapor pressure and that obtained from the correlation above.

$A := 69.006 \qquad B := -5599.6 \qquad C := -7.0985 \qquad D := 6.2237 \cdot 10^{-6} \qquad VP :=$

$$Psat(t) := \exp\left[A + \frac{B}{\dfrac{t}{K}} + C \cdot \ln\left(\frac{t}{K}\right) + D \cdot \left(\frac{t}{K}\right)^2\right] Pa$$

$T_{expr} := VP^{\langle 0 \rangle} \cdot K$

$P_{expr} := VP^{\langle 1 \rangle} \cdot K$

|   | 0 | 1 |
|---|---|---|
| 0 | 259.18 | $4.267 \cdot 10^3$ |
| 1 | 265.04 | $6.005 \cdot 10^3$ |
| 2 | 271.76 | $8.691 \cdot 10^3$ |
| 3 | 275.12 | $1.038 \cdot 10^4$ |
| 4 | 278.64 | $1.242 \cdot 10^4$ |
| 5 | 282.23 | $1.484 \cdot 10^4$ |
| 6 | 285.62 | $1.748 \cdot 10^4$ |
| 7 | 293.87 | $2.554 \cdot 10^4$ |
| 8 | 302.43 | $3.691 \cdot 10^4$ |
| 9 | 311.75 | $5.367 \cdot 10^4$ |
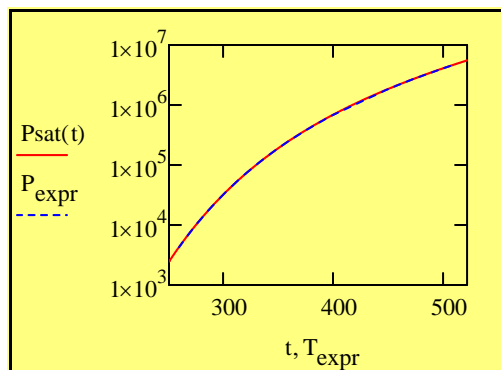


Notice that when I first try to create the plot, I don't get anything.  This is because I need to change the bounds.  Looking at the data, the temperature range should be from about 250 to 520 K.  Changing the bounds to this fixes the problem.

## Vapor Pressure of Acetone

Saturated Vapor Pressure (atm)

$\dfrac{Psat(t)}{atm}$

$\dfrac{P_{expr}}{atm}$

100

10

1

0.1

0.01

300     400     500

$t, T_{expr}$

Temperature (K)

——— Predicted

O O Experimental
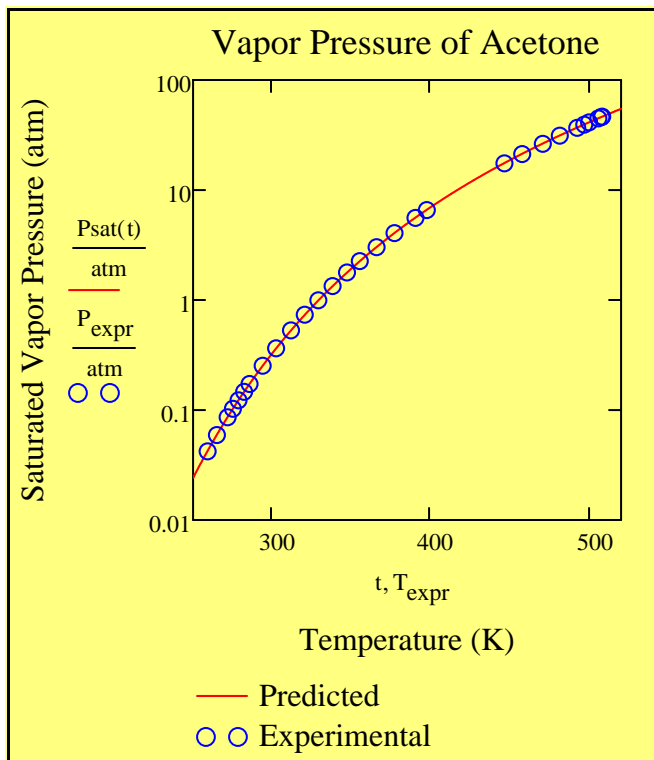
Key Points:
1. Notice that you can place the y-axis on a log scale by double clicking on the graph and checking the appropriate box.
2. Notice How you can the units on the graph by dividing the y arguments by the desired units.

## *Troubleshooting Tips/Don't Forget This*

Beginners in Mathcad often have trouble graphing.  Keep in mind the following.

1. Many of the problems with graphing can be solved by typing the correct bounds, even if you think they are correct to begin with.
2. Sometimes the problem is that you are using an independent variable that has been defined previously.  Try changing the variable name to something different.
3. Make sure that the argument type matches.
4. Make sure the units are correct.  The units of each y-axis argument must be the same.

## 2. Plotting to help you solve for roots of an equation.

Engineers solve equations everyday.  One of the first things you should always do when trying to find the roots of an expression is to create a plot.  This will tell you how many real roots the solution has as well as give you a good idea of where they are.

### *Demonstration*

For     $y = q^6 + 3q^5 - 5q^4 + 2 \cdot q^2 - 3 \cdot q + 1$   plot y vs q.  How many real roots are present?

$y(q) := q^6 + 3q^5 - 5q^4 + 2 \cdot q^2 - 3 \cdot q + 1$
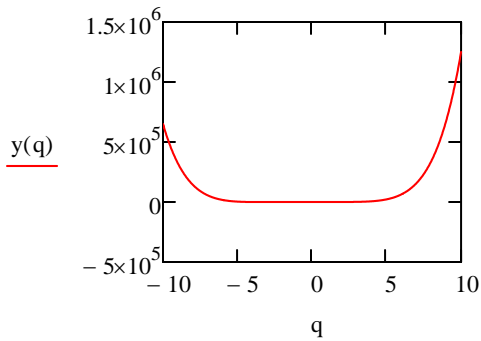
Try changing the bounds to make sure you get all the roots.



Key Points:
1. Since the roots of the equation are when the function equals zero, make the y-axis close to zero.
2. Zooming in reveals roots you can't initially see.
3. You can place a "marker" (or straight line) by double clicking on the graph, checking "Show markers", clicking OK, and then filling in the place holder that has appeared.
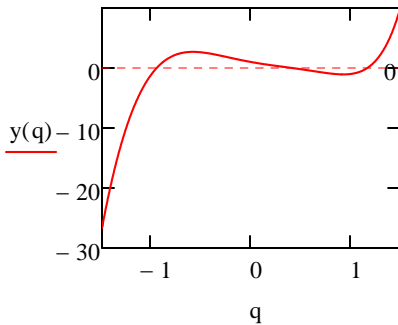4. The Trace utility, on the Graph tool palette, is useful to determine precise values for roots. (you must click on a graph to enable the Trace button.)

# 3. Differential Calculus

Differential Calculus is fairly simple using Mathcad.  The functions are found on the Calculus tool palette.

## A. Demonstration of operator syntax

First Order

$$\frac{d}{dx}x^3 \qquad \frac{d}{dx}f(x)$$

Higher Order

$$\frac{d}{dx}\frac{d}{dx}x^3 \qquad \text{or} \qquad \frac{d^2}{dx^2}x^3$$

Note, you can remove the ()'s the Mathcad adds between the derivative operators.

Mixed Derivatives

$$\frac{d}{dx}\frac{d}{dy}\left(x^2 + y^2\right)$$

$$\frac{d^2}{dx^2}\frac{d}{dy}\left(x^3 - y^2\right) \qquad \text{or} \qquad \frac{d}{dx}\frac{d}{dx}\frac{d}{dy}\left(x^3 - y^2\right)$$

## B. Numerical Evaluation of Derivatives

Explanation

To determine the numerical value of the derivative at some point, define the value of the independent variable, type the derivative, and then type =.

Demo

Find the value of the derivative of $x^2 + 2x + 3$ when x = 1.

$$f(x) := x^2 + 2 \cdot x + 3 \qquad d := 1 \qquad \frac{d}{dd} f(d) = 4$$

$$p := 1 \qquad \frac{d}{dp}\left(p^2 + 2 \cdot p + 3\right) = 4$$

Practice: Find the value of the fourth derivative of [2 sin(3x) + 4 cos(5x)] when x = 15°

$$f(x) := 2 \cdot \sin(3 \cdot x) + 4 \cdot \cos(5 \cdot x) \qquad x := 15 deg$$

$$\frac{d^4}{dx^4} f(x) = 761.599$$

Practice: Find the cross derivative d²/dxdy of $x^3 + 3 \cdot x^2 \cdot y + 3 \cdot x \cdot y^2 + y^3$ at x = 1 and y = 1.

$$f(x,y) := x^3 + 3 \cdot x^2 \cdot y + 3 \cdot x \cdot y^2 + y^3 \qquad x := 1 \qquad y := 1$$

$$\frac{d}{dx}\frac{d}{dy} f(x,y) = 12$$

## C. Symbolic Evaluation of Derivatives

Explanation: There are three ways to get a symbolic derivative
1. Use the arrow on the symbolic tool palette
2. Evaluate/Symbollically (on the Symbolics pulldown menu)
3. Variable/Differentiate (on the Symbolics pulldown menu)

Demo Method 1: the Symbolic toolbar

$$\frac{d}{dx}\left(x^2 + 2 \cdot x + 3\right) \rightarrow 4$$

$$f(x) := x^2 + 2 \cdot x + 3$$

$$\frac{d}{dx} f(x) \rightarrow 4$$

Not what you want.  Remember that the tool palette honors all previous calculations.

Demo Method 2, the Symbolics/Evaluate/Symbolically

$x := 1$

$\dfrac{d}{dx}\left(x^2 + 2 \cdot x + 3\right) = 4$      $\dfrac{d}{dx}\left(x^2 + 2 \cdot x + 3\right) \rightarrow 4$      Not what you want. Remember that the tool palette honors all previous calculations.

$\dfrac{d}{dx}\left(x^2 + 2 \cdot x + 3\right)$   yields   $2 \cdot x + 2$

Demo Method 3, the Symbolics/Variable/Differentiate

$x := 1$

$x^2 + 2 \cdot x + 3$   by differentiation, yields   $2 \cdot x + 2$      Note: to use this method remember to only select the variable by which you want to differentiate.

Key Points:
1. Remember, there is a slight difference between using the Tool Palette and using the pull down menu
2. The Tool Palette honors all previous calculations and will re-evaluate every time you change something in the worksheet.
3. The Pull down menu ignores any previously-defined variables/functions and does the calculation once.
4. Remember to select the entire expression when using method 2.
5. Remember to select only the variable of integration when using method 3.

Practice: Find the cross derivative, d²/dxdy of   $x^3 + 3 \cdot x^2 \cdot y + 3 \cdot x \cdot y^2 + y^3$

$\dfrac{d}{dx}\dfrac{d}{dy}\left(x^3 + 3 \cdot x^2 \cdot y + 3 \cdot x \cdot y^2 \, y^3\right)$   yields   $15 \cdot y^4 + 6 \cdot x$

# 3. Integral Calculus

Integral calculus straightforward in Mathcad. The symbols for both definite and indefinite integrals are found on the calculus tool palette.

## A. Demonstration: Numerical Evaluation of Definite Integrals

Integrate -4x² + 3x + 12 from -1 to 2.

$\displaystyle\int_{-1}^{2} -4 \cdot x^2 + 3 \cdot x + 12 \, dx = 28.5$

$$f(s) := -4 \cdot s^2 + 3 \cdot s + 12$$

$$\int_{-1}^{2} f(t)\, dt = 15$$

Integrating with units:

In engineering, we often have to integrate the heat capacity from one temperature to another to determine the heat required for a particular process. Mathcad can do the integration with the units included.

$$Cp(T) := \left(10 + 5 \cdot \frac{T}{K}\right) \cdot \frac{J}{mol \cdot K}$$

$$T_1 := 300K \qquad T_2 := 400K$$

$$q := \int_{T_1}^{T_2} Cp(t)\, dt = 1.76 \times 10^5 \cdot \frac{J}{mol}$$

## B. Multiple Integrals

The most difficult thing about multiple integrals is determining which bounds correspond to which variable of integration. For example, if I needed to do a triple integral, and I place three integral symbols in Mathcad, it would look like this.

$$\int_{\blacksquare}^{\blacksquare} \int_{\blacksquare}^{\blacksquare} \int_{\blacksquare}^{\blacksquare} \blacksquare \, d\blacksquare \, d\blacksquare \, d\blacksquare$$

It is often difficult to remember if you fill in the integral signs from the inside out or from left to right. To make sure that you don't make a mistake, the best way to do this is place one integral sign, fill it in, and then proceed to the next. For example, if I have a function $x^2 + y^2 + z$ that I must integrate with $0 < x < 1$, $-4 < y < 10$, and $-2 < z < 5$, I would proceed using the following steps.

$$\int_0^1 x^2 + y^2 + z\, dx$$

$$\int_{\blacksquare}^{\blacksquare} \int_0^1 x^2 + y^2 + z\, dx\, d\blacksquare$$

$$\int_{-4}^{10} \int_0^1 x^2 + y^2 + z\, dx\, dy$$

$$\int_{\blacksquare}^{\blacksquare} \int_{-4}^{10} \int_0^1 x^2 + y^2 + z\, dx\, dy\, d\blacksquare$$

$$\int_{-2}^{5} \int_{-4}^{10} \int_0^1 x^2 + y^2 + z\, dx\, dy\, dz = 2.662 \times 10^3$$

Practice the example given above.

$$\int_{-2}^{5}\int_{-4}^{10}\int_{0}^{1} x^2 + y^2 + z \, dx \, dy \, dz = 2.662 \times 10^3$$

## C. Symbolic Integrals

Explanation: As with differentiation, there are three ways to calculate a symbolic integral
1. Use the arrow on the symbolic tool palette
2. Evaluate/ Symbolically  (on the Symbolics pulldown menu)
3. Variable/Integrate (on the Symbolics pulldown menu)

**Indefinite Integrals**          Symbolically integrate     $x \cdot \ln(x) + \sin(x)$

Demo Method 1: the Symbolic toolbar

$x = 1$  Notice that x is already defined so I have to change the variable of integration.

$$\int b \cdot \ln(b) + \sin(b) \, db$$

$$\int b \cdot \ln(b) + \sin(b) \, db \rightarrow \frac{b^2 \cdot \ln(b)}{2} - \cos(b) - \frac{b^2}{4}$$

Demo Method 2, the Symbolics/Evaluate/Symbolically

$$\int x \cdot \ln(x) + \sin(x) \, dx \qquad \boxed{\text{yields}} \qquad \frac{x^2 \cdot \ln(x)}{2} - \cos(x) - \frac{x^2}{4}$$

Demo Method 3, the Symbolics/Variable/Differentiate

$$x \cdot \ln(x) + \sin(x) \qquad \boxed{\text{by integration, yields}} \qquad \frac{x^2 \cdot \ln(x)}{2} - \cos(x) - \frac{x^2}{4}$$

Key Points:
1. Remember that method 1 honors all previous calculations.
2. Remember to select the entire expression when using method 2.
3. Notice that no integral sign is needed for method 3.

**Definite Integrals**

You can symbolically evaluate definite integrals

$$\int_{a}^{b} -4 \cdot x^2 + 3 \cdot x + 12 \, dx \rightarrow \frac{4 \cdot a^3}{3} - \frac{3 \cdot a^2}{2} - 12 \cdot a - \frac{4 \cdot b^3}{3} + \frac{3 \cdot b^2}{2} + 12 \cdot b$$