Mathcad Lecture #3 In-class Worksheet
Functions - Solutions

At the end of this lecture, you should be able to:
- find and use intrinsic Mathcad Functions for fundamental, trigonometric, and if statements
- construct compound if functions using AND, OR, XOR, NOT operators
- create user-defined functions
- construct use-defined functions with appropriate units
- decide when a user-defined function is needed

1. Intrinsic Mathcad Functions

- Mathcad has a myriad of mathematical functions.
- You have already used several of them which are available on the calculator tool palette.
- Several common functions are found in the table to the right.
- You can see all the functions available, as well as a description of each, by clicking on **f(x)** button

Practice

1. Calculate the following: floor of π, ceil of π, the remainder of dividing π by 3.

$$\mathrm{floor}(\pi) = 3 \qquad \mathrm{ceil}(\pi) = 4 \qquad \mathrm{mod}(\pi, 3) = 0.142$$

2. Round π to 5 decimal places.

$$\mathrm{round}(\pi, 5) = 3.14159 \qquad \text{Note: you must change the display precision}$$

3. Let x = 0.9 and calculate the following: e^x, $\log_{10}(x)$, $\log_2(x)$, $\ln(x)$

$$x := 0.9 \qquad e^x = 2.46 \qquad \log(x) = -0.046 \qquad \log(x, 2) = -0.152 \qquad \ln(x) = -0.105$$

Trigonometric Functions

- All trigonometric function are available and are obtained by typing the name followed by ()'s.
- x is assumed to be in radians unless specified otherwise.

$$\sin(30) = -0.988 \qquad \sin(30\mathrm{rad}) = -0.988 \qquad \sin(30\mathrm{deg}) = 0.5 \qquad \sin\left(\frac{\pi}{6}\mathrm{rad}\right) = 0.5$$

2. User-Defined Functions

One of the powerful features of Mathcad is the ability to define your own functions. This is very helpful when doing engineering calculations and will be used often.

Syntax

To create a function:
1. Type the desired function name
2. Type (x) where x is the variable of the function.
3. Type : to give you :=
4. Define the expression in terms of x.

Note: you can use more then one variable in a function, simply separate each with a comma.

Demonstration

$$R_g := 8.314 \frac{J}{mol \cdot K} \qquad v_{ig}\left(N_{moles}, Temp, Press\right) := \frac{N_{moles} \cdot R_g \cdot Temp}{Press} \qquad v_{ig}(1mol, 273.15K, 1bar) = 22.71 \ L$$

Important Point 1

The parameter names in the argument list of the function are really just place holders. When I evaluate the function, I can use other variables which contain the numerical values I want to use in evaluation.

$\text{lbmol} := 453.59237 \text{mol}$

$n := 2\text{lbmol}$

$T_1 := 700R$ $\qquad$ $P_1 := 50\text{psi}$ $\qquad$ $v_{ig}(n, T_1, P_1) = 8.508 \cdot m^3$

$T_2 := 800R$ $\qquad$ $P_2 := 70\text{psi}$ $\qquad$ $v_{ig}(n, T_2, P_2) = 6.946 \cdot m^3$

Important Point 2

If you pass a function variables or numbers with incorrect units, the function will either not evaluate or return incorrect units.

$T_3 := 900$ $\qquad$ $P_3 := 80\text{psi}$ $\qquad$ $v_{ig}(n, T_3, P_3) = 12.307 \dfrac{m^3}{K}$

3. Creating Functions Which Require Specific Units

In engineering, we often need physical properties, such as heat capacities, to do design calculations. We usually obtain these by looking up an equation in a handbook. For example, the heat capacity of liquid water is dependent upon the temperature and can be found using the following equation.

$$\frac{Cp(T/_{\circ F})}{BTU/_{lbmol \cdot {}^\circ F}} = 18.4613 - 0.013584T + 7.24045 \cdot 10^{-5} T^2 - 1.87335 \cdot 10^{-7} T^3 + 2.14815 \cdot 10^{-10} T^4$$

The T/°F of the equation means the temperature must be in units of °F. The BTU/(lbmol*°F) means the heat capacity returned by the equation is in those units. Our challenge is to write a Mathcad function that always returns the correct heat capacity based upon the equation given above.

Before creating the function lets look at how Mathcad handles units and unit conversions.

$300K = 540 \cdot R$ $\qquad\qquad$ ASK

$\dfrac{300K}{R} = ?$ $\qquad\qquad$ "If you believe the top expression, what will this give you?"

$\dfrac{300K}{R} = 540$ $\qquad\qquad$ "What does 540 represent?"
Ans: The numerical value of the temperature in units of Rankine

$\dfrac{300K}{R} - 459.67 = 80.33$ $\qquad\qquad$ "What does 80.33 represent?"
Ans: The numerical value of the temperature in units of °F

Demonstration

$A := 18.4613 \quad B := -0.01358 \quad C := 7.24045 \cdot 10^{-5} \qquad D := -1.87335 \cdot 10^{-7} \qquad E := 2.14815 \cdot 10^{-10}$

$Cp(t) := A + B \cdot t + C \cdot t^2 + D \cdot t^3 + E \cdot t^4$

$Cp(300K) = \blacksquare$

$Cp(300) = 17.586$

ASK: "What is the correct value of the heat capacity?"

$Cp(540R) = \blacksquare$

$Cp(540) = 21.009$

If you don't place units in, the function doesn't know what units the temperature is in.

$Cp[(80.33 + 459.67) \cdot R] = \blacksquare$

$Cp(80.33) = 17.749$

$Cp(t) := A + B \cdot \left(\dfrac{t}{R} - 459.67\right) + C \cdot \left(\dfrac{t}{R} - 459.67\right)^2 + D \cdot \left(\dfrac{t}{R} - 459.67\right)^3 + E \cdot \left(\dfrac{t}{R} - 459.67\right)^4$

ASK: "What is the term in ()'s doing?"

$Cp(300K) = 17.749$

$Cp(300) = \blacksquare$

$Cp(540R) = 17.749$

$Cp(540) = \blacksquare$

We write the expression to force the number passed to the function to always be in °F.

$Cp[(80.33 + 459.67) \cdot R] = 17.749$

$Cp(80.33) = \blacksquare$

$Cp(t) := \left[A + B \cdot \left(\dfrac{t}{R} - 459.67\right) + C \cdot \left(\dfrac{t}{R} - 459.67\right)^2 + D \cdot \left(\dfrac{t}{R} - 459.67\right)^3 + E \cdot \left(\dfrac{t}{R} - 459.67\right)^4 \right] \cdot \dfrac{BTU}{lbmol \cdot R}$

$Cp(300K) = 74.314 \dfrac{m^2 \cdot kg}{mol \cdot K \cdot s^2}$

$Cp(300K) = 17.749 \cdot \dfrac{BTU}{lbmol \cdot R}$

By adding the units to the entire expression, we have made full use of Mathcad's unit capability. We can now use the Cp function in later calculations and all units will be correct.

$Cp(540R) = 74.314 \dfrac{m^2 \cdot kg}{mol \cdot K \cdot s^2}$

$Cp(540R) = 17.749 \cdot \dfrac{BTU}{lbmol \cdot R}$

$Cp[(80.33 + 459.67) \cdot R] = 74.314 \dfrac{m^2 \cdot kg}{mol \cdot K \cdot s^2}$

$Cp[(80.33 + 459.67) \cdot R] = 17.749 \cdot \dfrac{BTU}{lbmol \cdot R}$

Key Points:
- The term in ()'s forces Mathcad to convert any temperature passed to the function into °F as required by the original expression.
- Multiplying the entire expression by the correct units allows us to use the function in subsequent calculations and let Mathcad do the unit conversions.

Practice

The Reynold's number is a **dimensionless** quantity used to characterize fluid flow. For a circular pipe, the Reynold's number is defined as $\text{Re} = \dfrac{\rho D v}{\mu}$, where ρ and μ are the density and viscosity of the fluid, D is the diameter of the pipe, and v is the velocity of the fluid through the pipe. Calculate the Reynold's number if water, at 2 degrees Celsius, travels down a 2 in diameter pipe at 3 m/s. The density and viscosity of water can be found from the expressions below. The molecular weight of water is 18.02 gm/mol.

$$\frac{\mu\left(\dfrac{T}{K}\right)}{Pa \cdot s} = \exp\left[-52.843 + \frac{3703.6}{T} + 5.8660\ln(T) - 5.879 \cdot 10^{-29} T^{10.0000} \right]$$

$$\frac{\rho\left(T / {}^\circ C\right)}{lbmol \cdot ft^{-3}} = 7.98187 + 0.00138771T + 2.87476 \cdot 10^{-6} T^2 - 1.94845 \cdot 10^{-9} T^3 + 3.72428 \cdot 10^{-13} T^4$$

$A1 := -52.843$ $B1 := 3703.6$ $C1 := 5.8660$ $D1 := -5.879 \cdot 10^{-29}$

$A2 := 7.98187$ $B2 := 0.00138771$ $C2 := 2.87476 \cdot 10^{-6}$ $D2 := -1.94845 \cdot 10^{-9}$ $E2 := 3.72428 \cdot 10^{-13}$

$MW := 18.02 \dfrac{gm}{mol}$ $v := 3 \dfrac{m}{s}$ $D := 2in$

$$\mu(t) := \exp\left[A1 + \frac{B1}{\dfrac{t}{K}} + C1 \cdot \ln\left(\frac{t}{K}\right) + D1 \cdot \left(\frac{t}{K}\right)^{10} \right] \cdot Pa \cdot s$$

$$\rho(t) := \left[A2 + B2 \cdot \left(\frac{t}{K} - 273.15\right) + C2 \cdot \left(\frac{t}{K} - 273.15\right)^2 + D2 \cdot \left(\frac{t}{K} - 273.15\right)^3 + E2 \cdot \left(\frac{t}{K} - 273.15\right)^4 \right] \cdot \frac{lbmol}{ft^3}$$

$T := (2 + 273.15) \cdot K$

$$Re := \frac{\rho(T) \cdot D \cdot v}{\mu(T)} \cdot MW \qquad Re = 2.182 \times 10^5$$

4. The if function

Syntax

if(*condition, true, false*)
- True and false can be functions, text, letters, numbers, etc.
- Different conditions can be found on the Boolean tool palette.

Demonstration

$absval(x) := if(x \geq 0, x, -x)$ $absval(-3) = 3$ $absval(3) = 3$

Compound Logical Expressions

Operator	Symbol on Tool Palette	Description
And	∧	Returns 1 if both x and y are non-zero (true). Returns 0 otherwise.
Or	∎ ∨ ∎	Returns 1 if either x or y are non-zero (true). Returns 0 otherwise.
Exclusive or, xor	∎ ⊕ ∎	Returns 1 if either x or y, but not both, are non-zero (true). Returns 0 otherwise.
Not	¬∎	Returns 1 if x is zero (false). Returns 0 otherwise
Less than	∎ < ∎	Returns 1 if x < y, 0 otherwise
Greater than	∎ > ∎	Returns 1 if x > y, 0 otherwise
Equal to	∎ = ∎	Returns 1 if x + y, 0 otherwise
Not Equal to	∎ ≠ ∎	Returns 1 if x does not equal y, 0 otherwise

Demonstration

Imagine that the temperature of the reactor must be kept above 170 °C to keep the reaction going and below 200 °C to prevent excessive pressure from forming in the vessel. Write a simple logical function to determine if the reaction temperature is in the correct range.

$check\_reactor(t) := if\left[(t > 200\,°C) \lor t < 170\,°C, "help", "reactor\ is\ OK"\right]$

$check\_reactor(180\,°C) = "reactor\ is\ OK"$

$check\_reactor(201\,°C) = "help"$

$check\_reactor(100\,°C) = "help"$

Practice

Write a logical function to turn on a drainage pump if the liquid height in a holding tank exceeds 3.1 *meters* <u>and</u> inlet flow rate exceeds 24.5 *gal/min*. If both of these conditions are met, the output from the function should equ 1.0, otherwise the output should equal 0 (zero).

$$\text{drain}(h, f) := \text{if}\left(h > 3.1m \wedge f > 24.5\frac{\text{gal}}{\text{min}}, 1, 0\right)$$

$$\text{drain}\left(5m, 25\frac{\text{gal}}{\text{min}}\right) = 1 \qquad \text{drain}\left(3m, 25\frac{\text{gal}}{\text{min}}\right) = 0 \qquad \text{drain}\left(5m, 24\frac{\text{gal}}{\text{min}}\right) = 0$$

$$\text{drain}\left(3m, 24\frac{\text{gal}}{\text{min}}\right) = 0$$

Extra Practice

For x between 0 and 1, the series expansion of ln(1+x) is

$$\ln(1 + x) = \sum_{i=1}^{n} (-1)^{i+1} \frac{x^i}{i}$$

The series is exact if n = ∞, but such a computation is computationally impossible.

Construct a function named ln1x which accepts two parameters (x and n) and calcualtes ln(1+x) according to the seried above. Do a small parametric study to determine now many terms are needed to obtain accuracy out to the 6th decimal place. Check the results against the instrinsic MatCAD function.

$$\text{ln1x}(x, n) := \sum_{i=1}^{n} \left[(-1)^{i+1} \cdot \frac{x^i}{i}\right]$$

$$\ln(1 + 0.5) = 0.405465$$

$\text{ln1x}(0.5, 1) = 0.500000$	$\text{ln1x}(0.5, 11) = 0.405479$
$\text{ln1x}(0.5, 3) = 0.416667$	$\text{ln1x}(0.5, 13) = 0.405468$
$\text{ln1x}(0.5, 5) = 0.407292$	$\text{ln1x}(0.5, 15) = 0.405466$
	$\text{ln1x}(0.5, 16) = 0.405465$
$\text{ln1x}(0.5, 7) = 0.405804$	$\text{ln1x}(0.5, 17) = 0.405465$
$\text{ln1x}(0.5, 9) = 0.405532$	$\text{ln1x}(0.5, 19) = 0.405465$

Functions	Description
$\exp(x)$, e^x	The number e raised to the power of x.
$\log(x,[b])$	Base b logarithm of x. (b=10 by default)
$\ln(x)$	Natural logarithm of x
$x!$	Factorial of x.
max(a,b,c,…)	The value of the largest argument.
min(a,b,c,…)	The value of the smallest argument.
mean(a,b,c,…)	The average of a,b,c,…
floor(x)	The largest integer less than or equal to x.
ceil(x)	The smallest integer greater than or equal to x.
round(x,n)	The number x rounded to n decimal places. If n is omitted, x is rounded to the nearest integer. If n<0, x is rounded to the lefts of the decimal point.
trunc(x)	The integer part of x
mod(x,y)	The reminder of dividing x by y
if(*condition,true,false*)	If *condition* is true (non-zero), returns *true*. If *condition* is false, returns *false*.
(calculus toolbar)	$\sum_{i=1}^{n} f(i)$, summation
(calculus toolbar)	$\prod_{i=1}^{n} f(i)$, product